Investigate the efficiency of Proposed Techniques To Improve Area Calculation Using Simpson And Trapezoidal Rules

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Key words: Area calculation; Simpson’s rule; Trapezoidal rule; Circular segment; Parabola.

SUMMARY

The trapezoidal rule and Simpson’s rule are numerical approximation methods to be used to approximate the area under a curve. The area is divided into (n) equal pieces, called a subinterval or trapezoid. Each subinterval is approximated as a trapezoid considering the outer edge as straight line in the trapezoidal rule. In Simpson’s rule, each two subintervals approximated as a trapezoid and a parabola.
This paper provides two techniques as trails to improve the area calculated using Simpson’s rule and trapezoidal rule. The first proposed technique deals with the curved part as a circular segment instead of parabola in Simpson’s rule. The second technique add or subtract a small parabola to the calculated area when using trapezoidal rule.
The proposed techniques were applied on several numerical examples of known area and the results were compared.

ملخص:

تعتبر طريقة سمبسون وطريقة أشباه المنحرفات من الطرق الحسابية الشهيرة المستخدمة لتقدير المساحة تحت المنحنى في حالة عدم القدرة على تكامل معادلة المنحنى أو في حالة استخدام قياسات عند نقط متفرقة على المنحنى. ويتم تقدير المساحة بتقسيمها إلى أجزاء صغيرة وحساب مساحة كل جزء على حده على انه شبه منحرف أو كل جزأين على أنهما شبه منحرف يعلوه قطع مكافئ ثم تجميع مساحات هذه الأجزاء معا للحصول على المساحة الكلية.
وهذا البحث يعرض طريقتين لحساب المساحات، الأولى باعتبار الحد الخارجي لكل جزأين عبارة عن قوس دائري وحساب مساحة الجزء الذي يعلو شبه المنحرف على انه قطعة دائرية، والطريقة الثانية باستخدام طريقة أشباه المنحرفات وإضافة أو طرح المساحة المحصورة بين الحد الخارجي والحد العلوي لشبه المنحرف وقد تم تطبيق الطريقتين المقترحتين على عدة أمثلة حسابية ثم مقارنة النتائج.

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1. INTRODUCTION

Computing the area of figures of irregular boundaries is an interested and important subject in surveying. These computations are encountered when surveying shores, ponds hills and irregular properties (Fouad 1983). Numerical integration is used to compute the area because the data in such cases are discrete (Wellin J.). There are several methods that numerically approximate the area of such irregular figures. Two of these methods are well known, they are Trapezoidal rule and Simpson's rule.

Using both methods to approximate the area under a curve first involves dividing the area into a number (*n*) of subintervals of equal width (*b*) and then measuring the offsets (*h*) to the curved boundary as shown in figure (1). Trapezoidal rule which represented by equation (1) approximating the area of each subinterval by the area of the trapezium formed when the upper end is replaced by a chord. Simpson's rule shown in equation (2) works by approximating irregular border by the quadratic polynomial (parabola) which takes the same values of the offsets at three points (two subintervals) (e.g. h_0, h_1 and h_2 in figure (1)).

$$Area \cong \frac{b}{2}\sum_{k=1}^{n}\left(h_k + h_{k-1}\right) \qquad (1)$$

$$Area \cong \frac{b}{3}\sum_{k=1}^{n/2}\left(h_{2k-2} + 4h_{2k-1} + h_{2k}\right) \qquad (2)$$

Where *(n)* is the number of subintervals – (should be even for Simpson's rule).

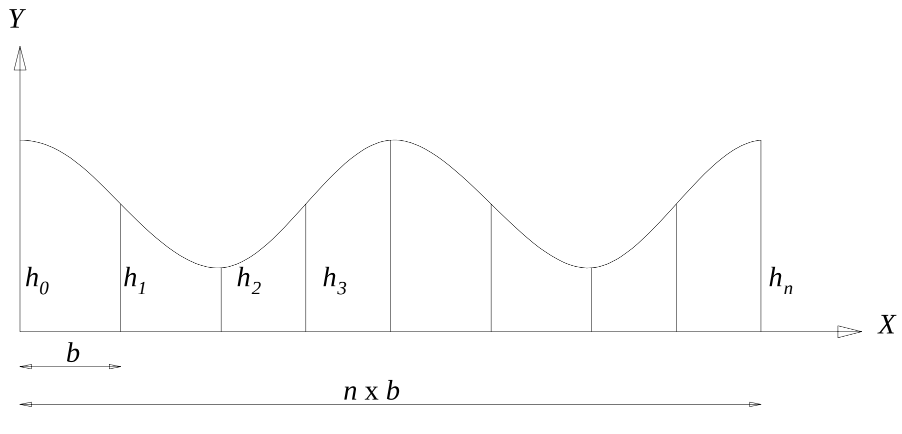


Figure (1): Offsets and subintervals

Fouad 1983 proposed a fifth degree polynomial equation used to compute the area of the middle interval of a section of five intervals. He generated a complicated equation for computing the area. He also proposed a third degree polynomial to be used for sections consisting of three intervals. The area under the curved boundary can be computed according to the third degree polynomial using the following equation:

$$Area \cong \frac{b}{24}\left[\begin{array}{l} a_1(h_0+h_n)+(a_2-P_n)(h_1+h_{n-1})+(a_3-P_{n-1})(h_2+h_{n-2}) \\ +(a_3+1)(h_3+h_4+\ldots+h_{n-3}) \end{array}\right] \quad (3)$$

Where n is the number od subintervals (minimum of 3)
$a_1 = 9$, $a_2 = 28$, $a_3 = 23$, $P_3 = 1$ and $P_i = 0$ (where $i \geq 4$)

Maling 1989 gave a detailed values for the equation parameters according to the number of intervals used in the calculation.
Elezovic and Pecaric 1998 and Cruz-Uribe and Neugebauer, 2002 and Ujevic 2004 studied the error in trapezoidal and Simpson's rule from a pure mathematical point of view.
This paper is an attempt to improve the area calculated using trapezoidal rule and invistigating the accuracy of calculating the area considering the outer borders of each two subintervals as a circular curve.

2. PROPOSED TECHNIQUES

2.1 The first technique

The first proposed technique deals with the upper part as a circular segment instead of parabola in Simpson's rule as shown in figure (2).

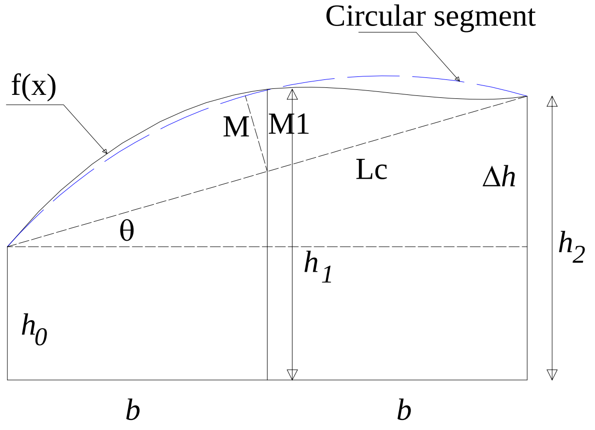


Figure (2): The circular segment

The radius (R)of the circular curve can be computed after (Khalil 2003) as:

$$R = \frac{Lc^2}{8M} + \frac{M}{2} \tag{4}$$

Where R: curve radius

$$M \cong M_1 \cos(\theta) \tag{5}$$

$$M1 = h_1 - \frac{h_0 + h_2}{2} \tag{6}$$

$$\theta = \tan^{-1} \Delta h / 2b \tag{7}$$

$$Lc = \sqrt{(\Delta h)^2 + (2b)^2} \tag{8}$$

$$\Delta h = h_2 - h_0 \tag{9}$$

The central angle (I) for this segment can be computed after (Khalil 2003) as:

$$Sin(I/2) = Lc/2R \tag{10}$$

The area of the circular segment can be computed as:

$$A_1 = \pi R^2 \frac{I}{360} - 0.5R^2 \sin(I) \tag{11}$$

The area under the curve for each two intervals is computed as:

$$A \cong b(h_0 + h_2) + A_1 \tag{12}$$

The total area for (n) even intervals is computed as:

$$Area \cong \sum_{k=1}^{n/2} \left[b\,(h_{2k-2} + h_{2k}) + A_k \right] \tag{13}$$

2.2 The second technique

The second technique add or subtract a small parabola to the calculated area using trapezoidal rule as shown in figure (3).

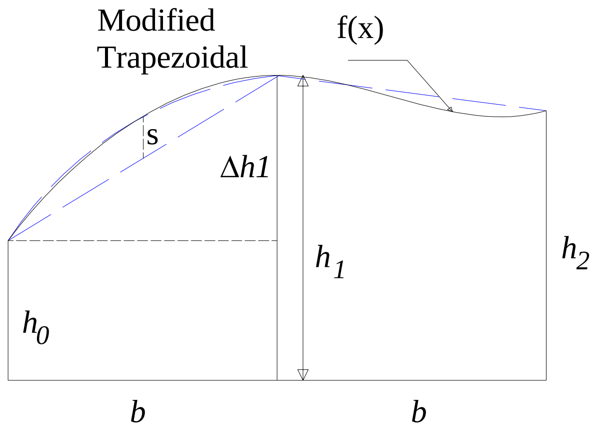


Figure (3): The added parabola to the trapezoid

The area of the parabola is computed as:

$$A_p = \frac{2}{3} b.s \quad (14)$$

$$s \cong \frac{\Delta h_1}{4} \quad (15)$$

$$\Delta h_1 = abs(h_1 - h_0) \quad (16)$$

The area under the curve for the first interval is computed as:

$$A_1 \cong \frac{b}{2}(h_0 + h_1) \pm A_{p_1} \quad (17)$$

The area of the parabola may be add or subtract according to the following criteria;
The area of the parabola will be add if h_1 is greater than the mean value of h_0 and h_2 (i.e. M1 is positive), the area of the parabola will be subtracted if M1 is negative. If the last offset was zero, the sign of the area of the parabola taken as the preceding interval. M1 is computed using equation (6).

The total area for (n) intervals is computed as:

$$Area \cong \sum_{k=1}^{n} \left[\frac{b}{2}(h_{k-1} + h_k) \pm A_{p_k} \right] \quad (18)$$

3. APPLICATION

The procedure to test the proposed techniques is to calculate the actual area under a certain function and compare it with the area estimated by different methods using the calculated offsets. The examples of Fouad (1983) were used to illustrate the efficiency of the proposed techniques. An excel sheet was prepared to calculate the area using different methods.

In the first example the actual area of half circle of radius 5, shown in figure (4) is calculated. Then it was divided into 10 intervals with equal spacing and the corresponding offsets were calculated. Five methods were applied to calculate the area by numerical integration, they are the first proposed technique, the second proposed technique, Simpson's rule, trapezoidal method and Fouad method. The error in the calculated area (difference from the actual area) and the percentage of this error (error/actual area) were calculated and the results are shown in table (1). Comparing the errors of the proposed techniques with those of the other methods it is clear that the proposed techniques give better.

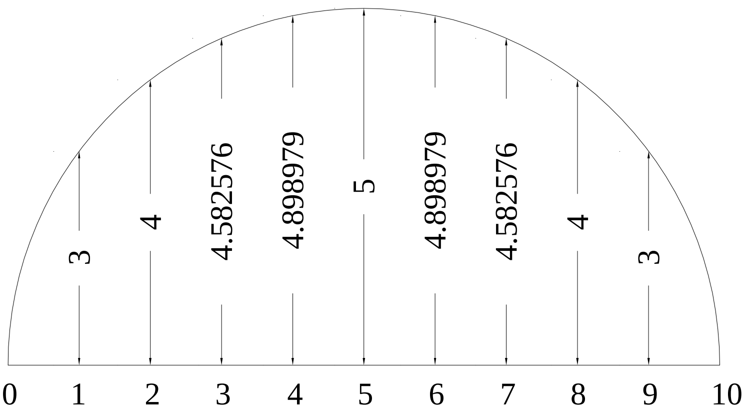


Figure (4): First numerical example

Table (1) The results of the 1st numerical example

Method (1)	Area (2)	*Error* (3)	*Error Percentage* (4)
Actual (A_a)	39.2699		
1st technique (A_r)	38.7745	-0.495	1.26
2nd technique (A_k)	39.6298	0.360	0.92
Simpson (A_s)	38.7522	-0.518	1.32
Trapezoidal (A_t)	37.9631	-1.307	3.33
Fouad (A_f)	38.6298	-0.640	1.63

In the second example the sin function was chosen since it resembles the configuration of natural curvilinears as mentioned by Fouad (1983), and it has points of curvature inflections. The area under the sin curve is calculated after shifting the *x*-axis downward by two units. The offsets at maximum and minimum were then calculated and the area under the curve from X = 0.5 π to X = 16.5 π was calculated as shown in figure (5).

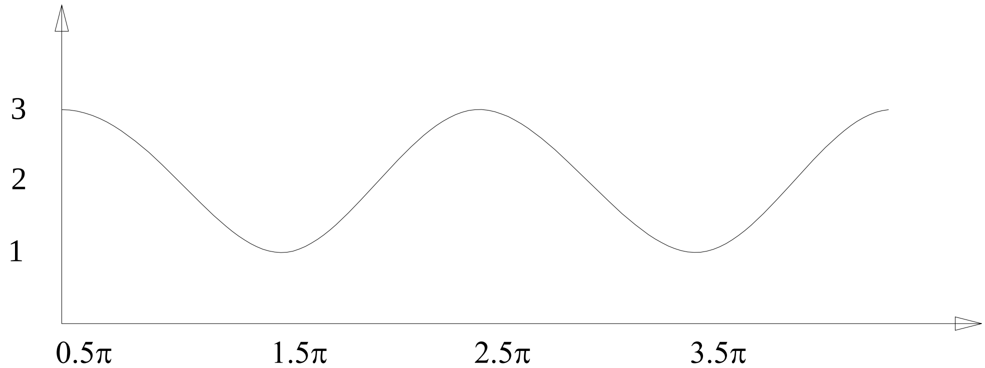


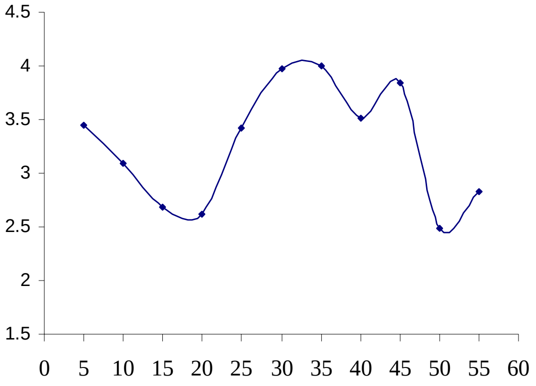
Figure (5): Second numerical example

The results shown in table (2) represent the area that calculated using only the offsets at the maximum and minimum points. The second proposed technique gave area exact as the actual, it reduces the error of about 17% than Simpson's rule. The first proposed technique gave an error of about 22%, this may be because of ignoring the inflection points. Trapezoidal method gave the exact area because the upper cord pass through the inflection point so it add and subtract the same amount of area from the actual area of each segment.

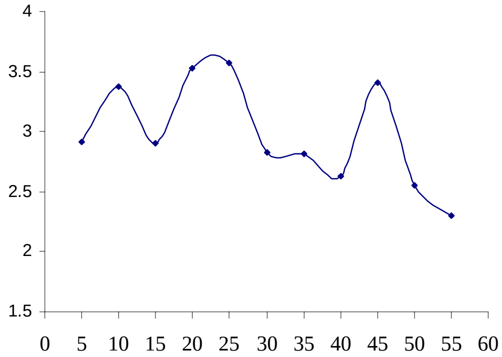
Table (2) The results of the 2nd numerical example (interval width = π)

Method (1)	Area (2)	*Error* (3)	*Error Percentage* (4)
Actual (A_a)	100.53096		
1st technique (A_r)	78.62180	-21.909	21.79
2nd technique (A_k)	100.53096	0.000	0.00
Simpson (A_s)	83.77580	-16.755	16.67
Trapezoidal (A_t)	100.53096	0.000	0.00
Fouad (A_f)	98.43657	-2.094	2.08

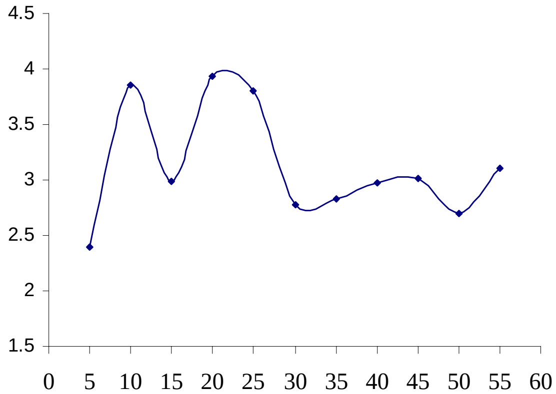
The offsets at the points of inflection were calculated and the area under the curve was calculated once again using an interval width of 0.5 π. In this case the two proposed



(a)



(b)



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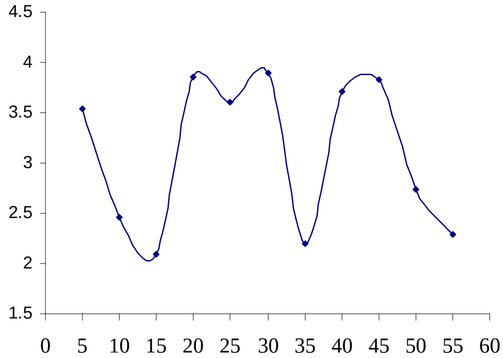

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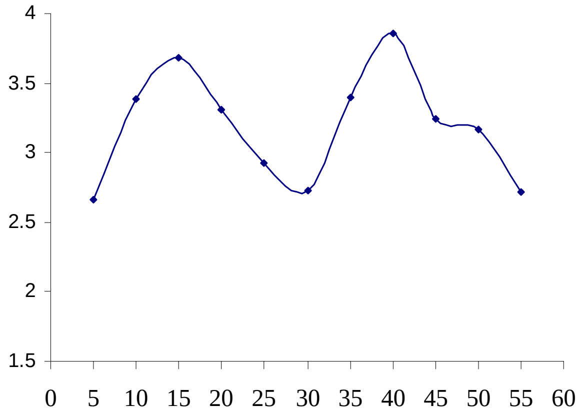
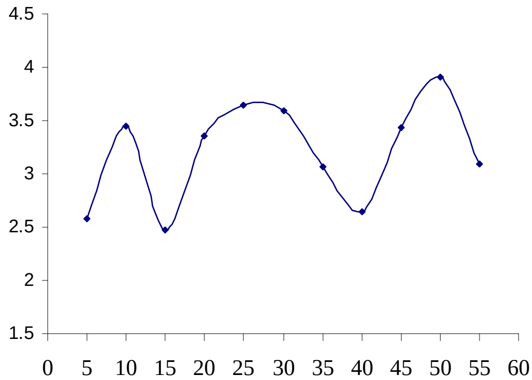


(e)



(f)

Figure (6) Third numerical example

techniques gave the exact area as the actual. So, the offsets at the points of inflection should be measured and used when calculating the area under irregular boundary specially if the first proposed technique was used.

The proposed thechniques were applied on several random examples shown in figure (6). The percentage of errors are shown in table (3). From which it is clear that the first proposed technique is as accurate as simpson's rule and the second proposed technique is as accurate as trapizoidal method.

Table (3) Percentage of error in area computations of third example

Method (1)	a (2)	*b* (3)	*c* (4)	*d* (5)	*e* (6)	*f* (7)	*Average*
1st technique (A_r)	2.07	0.76	0.08	1.89	0.32	1.21	**1.06**
2nd technique (A_k)	0.93	0.76	0.92	0.94	0.06	0.02	**0.60**
Simpson (A_s)	2.05	0.76	0.13	1.83	0.32	1.19	**1.04**
Trapezoidal (A_t)	0.65	0.46	0.91	0.28	0.33	0.29	**0.48**
Fouad (A_f)	1.01	0.17	0.46	0.63	0.12	0.51	**0.48**

4. CONCLUSIONS

This paper presents two proposed techniques to calculate the area under irregular curved boundary. The proposed techniques were applied on a numerical examples of Fouad 1983 and on several random examples. The results showed that the first proposed technique is as accurate as simpson's rule and the second proposed technique is as accurate as trapizoidal method. Fouad's method gave the accurate results. The proposed techniques nither improve the claculated area nor simplify the calculations.

REFERENCES

1- Burden, R. L. and Douglas F. J., 2000, Numerical Analysis, 7th Ed., Brooks/Cole. ISBN 0-534-38216-9.
2- Cruz-Uribe, D. and Neugebauer, C. J., 2002, Sharp Error Bounds for The Trapezoidal Rule and Simpson's Rule, Journal of Inequalities in Pure and Applied Mathematics, Vol. 3, Issue 4, Article 49.
3- Elezovic, N. and Pecaric, J., 1998, Stefenssen's Inequality and Estimates of Error in Trapezoidal Rule, Appl. Math. Lett. Vol. 11, No. 6, Pages 63-69.

4- Fouad, A., 1983, Optimization techniques in curved area calculation, Survey Review, Vol. 27, 210, Pages 163-168.
5- Khalil, R., 2003, Direct numerical method for calculating the parameters of horizontal circular curves, Civil Engineering Research Magazine (CERM), Al-Azhar University, Vol.25, No. 2, Cairo, Pages 580-586.
6- Maling, D.H., 1989, Measurements from maps, Pergamon press, London, UK.
7- Talman, L., 2006, Simpson's Rule Is Exact for Quintics, The Mathematical Association of America, [Monthly 113 February 2006], Pages 144-155.
8- Ujevic, N., 2004, Sharp Inequalities of Simpson Type and Ostrowski Type, Computers and Mathematics with Applications, Vol. 48, Pages 145-151.
9- Wellin, J. D., Numerical Integration, lecture notes, Available at: http://www.rit.edu/~jdweme/emem440/Numerical%20Integration.doc

OTHER READINGS

1- Anderson, J. M., Mikhail, E. M., Davis R. E. and Foote F. S., 1985, Introduction to surveying, McGraw-Hill Ryerson, New York, 1985.
2- Horwitz A., 2001, A version of Simpson's rule for multiple integrals, Journal of Computational and Applied Mathematics, 134 (2001), Pages1–11
3- Kavanagh, B. and Bird G., 2000, Surveying: principles and applications, 5th ed., Prentice-Hall, Inc., New Jersey, USA 2000.
4- Moffitt F. H. and Boucherd H., 1987, Surveying, 8th ed., Happer & Row publishers, New York, 1987.
5- Newton M., 2005, Trapezoidal Rule and Simpson's Rule, Calculator lesson 15 . available at: www.thiel.edu/mathproject/CalculatorLessons/L015.pdf
6- Stober P. and Yeh, Shi-Teo, 2002, An Explicit Functional Form Specification Approach to Estimate the Area under a Receiver Operating Characteristic (ROC) Curve, SUGI 27 proceedings, Paper 226, April 14-17, 2002. available at: www2.sas.com/proceedings/sugi27/p226-27.pdf
7- Wolf P. R. and Brinker R. C., 1989, Elementary Surveying, Harper & Row publishers, New York, 1989.
8- Yeh, Shi-Teo, 2002, Using Trapezoidal Rule for the Area Under a Curve Calculation, SUGI 27 proceedings, paper 229, April 14-17, 2002. available at: www2.sas.com/proceedings/sugi27/p229-27.pdf

BIOGRAPHICAL NOTES

Dr. Ragab Khalil graduated in 1989 from Civil Engineering Department, Faculty of Engineering, Assiut University, Egypt. In 1999 he awarded the Ph.D degree in Surveying through a cooperation program between Assiut University, Egypt and Innsbruck University, Austria. Now he is an assistant professor at Faculty of Environmental Design, King Abdulaziz University, Saudi Arabia.

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