THE OPTIMUM CHOICE FOR CONTOURING

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ملخص:

عند إجراء الرفع المساحى لعمل نماذج رقمية لسطح الأرض أو رسم خرائط كنتورية بدقة عالية تمثل سطح الأرض تمثيلا صحيحا ترد عدة تساؤلات أولها عن طريقة الرفع المثلى هل هى طريقة المربعات Regular Grid أو طريقة النقط المتفرقة Scattered points . وثانيها عن الطريقة المستخدمة فى عمل Interpolation وقد تم اختبار دقة تسع طرق لعمل Interpolation هى:

1) Inverse distance to a power, 2) Kriging, 3) Minimum curvature, 4) Modified Shepard's method, 5) Natural neighbour, 6) Nearest neighbour, 7) Polynomial regression, 8) Radial basis function and 9) Triangulation with linear interpolation.

والتساؤل الثالث هو هل توجيه حدود الأرض Orientation أو الميل عن الشمال هل له تأثير على طريقة الـ Interpolation وبالتالي على دقة الخرائط الكنتورية المنتجة. وهذا البحث يحاول الإجابة على هذه التساؤلات وقد تم إجراءه على مساحة ٤ هكتار من الجبال المحيطة بوادى منى فى مكة وذلك بعمل ترقيم Digitizing لخريطة للمنطقة المذكورة -حتى يمكن مقارنة النتائج بالخريطة الأصلية- والحصول على نقط فى شكل Regular Grid و Scattered points وذلك لدراسة Sample pattern كما تم عمل دوران للنقط المقاسة فى الحالتين بزوايا ١٥ ، ٣٠، ٤٥، ٦٠، ٧٥، ٩٠ درجة عن الشمال لدراسة تأثير Orientation وكذلك اختيار طريقة الـ Interpolation الأمثل من الطرق المذكورة.

ABSTRACT

The Digital Elevation Model (DEM) is an important part of mapping and is used for several purposes including contours derivation, image interpretation and several Geographic Information System (GIS) applications. Interpolation is often required to create DEM from sparse number of points. In this paper the interpolation accuracy of nine methods namely: 1) Inverse distance to a power, 2) Kriging, 3) Minimum curvature, 4) Modified Shepard's method, 5) Natural neighbour, 6) Nearest neighbour, 7) Polynomial regression, 8) Radial basis function and 9) Triangulation with linear interpolation are tested.

The effects of data pattern and orientation on choosing the proper interpolation method are studied. The study was applied on an area of 4 hectares of Makka mountains. A contour map of the area was digitized and a regular grid and scattered points samples were produced to study the pattern effect. The two samples were rotated by 15, 30, 45, 60, 75, and 90 degree from north to study the effect of the orientation. The resultant contour map for each study case was compared with the original map to choose the optimum interpolation method for each case.

KEY WORDS: Interpolation methods; DEM; Contour map; Pattern; Orientation, Accuracy.

INTRODUCTION

A Digital Elevation Model (DEM) consists of a pattern of data points of known horizontal coordinates (x, y) and heights (z) which represent a terrain surface (Algarni and Elhassan 2001). DEMs have a wide range of applications in surveying and mapping stated by (Mohamed at al 1996a and 1996b). They are also an essential part of a well-developed GIS, digital image rectification and orthophoto production. The DEM also aids automatic recognition of terrain features in town planning and automatic building extraction and offers automated assessment of land resources and attributes (Algarni and Elhassan 2001). DEMs have a major role to play in hydrological modeling, analysis of visibility and hazard mapping (Ardiansyah and Yokoyama 2002). DEM is very important to the validity of soil erosion model (Xie et al 2003). The DEM technique is becoming a powerful tool for representation of both existing and proposed ground surface in the fields of civil surveying, geology, mining engineering (Du 1996). DEMs have been used to delineate drainage networks and watershed boundaries, to calculate slope characteristics, to enhance distributed hydrologic models and to produce flow paths of surface runoff (Wang and Yin 1998).

There are different procedures and techniques for collecting the data to generate DEMs. DEMs are routinely created by digitizing contour maps, by direct field observations using ground surveying methods, by photogrammetric procedures and recently by laser profiling and laser scanning. Contour lines continue to be used to generate DEMs because in most centuries, this data covers the whole area in different scales, thus representing a cheap data source (Ardiansyah and Yokoyama 2002). Different patterns for creating DEMs from contours are studied by (Li 1994). Digitized contour lines can be obtained from maps by manual digitizing, semi-automated line following or automatic raster-scanning (Taud et al 1999). The manual digitizing was used in this research.

The sampling pattern for generating DEMs is an important aspect of the task. It may include: 1) regular mode, in which the spot heights are measured in a regular geometric pattern (gridded DEM), or 2) quasi-regular mode, in which the data are observed along parallel lines spaced at regular intervals but with data randomly spaced along each line, or 3) irregular mode, in which the data are collected at random without regard to their geometric distribution (Algarni and Elhassan 2001). The gridded DEM is the easiest of all to manipulate using machines, even though it is time consuming specially if ground survey is using for collect the data and it can misrepresent the real surface in areas of highly variable terrain. However, gridded DEMs can not always be created in the field, so regular and irregular pattern modes are tested in this paper. The orientation of the sampled data is a new parameter studied in this paper.

Interpolation is one of the most important parts of DEM building (Xie et al 2003). It can be defined as procedure of determining the height of any intermediate point of known planimetric coordinates. Interpolation produces a regularly spaced array of Z values from irregularly spaced XYZ data for contour or surface plots. The estimation of these points usually on a square lattice is known as gridding. Estimating

the value of properties at unsampled locations within the existing data points perimeter is known as interpolation, outside the perimeter defined by the data points extrapolation attempts to estimate values. Extrapolated values tend not to be as accurate as interpolated values. Estimates obtained from interpolation should reflect the real world physical features by incorporating spatial trends which are present in the point data (Wilson 1996). Evaluation of different interpolation methods are discussed by (Mohamed et al 1996b) and (Wood and Fisher 1993).
In this paper the interpolation accuracy of nine methods namely: 1) Inverse distance to a power, 2) Kriging, 3) Minimum curvature, 4) Modified Shepard's method, 5) Natural neighbour, 6) Nearest neighbour, 7) Polynomial regression, 8) Radial basis function and 9) Triangulation with linear interpolation are tested for the accuracy of the resultant contour maps These methods are available through the SURFER software, so the results of this paper may offer a guide for the users of this software.

METHODOLOGY

Interpolation methods are divided into two categories, exact and smoothing interpolators. Exact interpolators honor data points exactly when the point coincides with the grid node being interpolated. In other words, a coincident point carries a weight of essentially 1.0 and all other data points carry a weight of essentially zero. Smoothing interpolators or smoothing factors can be employed during gridding when you do not have strict confidence in the repeatability of your data measurements. This type of interpolation reduces the effects of small-scale variability between neighboring data points.
A brief review of the mathematical background of the nine interpolation methods used in this research are given below.

Inverse distance to a power

The Inverse Distance to a Power gridding method is a weighted average interpolator, and can be either an exact or a smoothing interpolator. With Inverse Distance to a Power, data are weighted during interpolation such that the influence of one point relative to another declines with distance from the grid node. The equation used for Inverse Distance to a Power is (SURFER manual):

$$\hat{Z}_j = \frac{\sum_{i=1}^{n} \frac{Z_i}{h_{ij}^{\beta}}}{\sum_{i=1}^{n} \frac{1}{h_{ij}^{\beta}}} \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$h_{ij} = \sqrt{d_{ij}^{2} + \delta^{2}} \quad \ldots\ldots\ldots\ldots \quad (2)$$

where:

n is the number of scattered points of the set;
h_{ij} is the effective separation distance between grid node "j" and the neighboring point "i."

$\hat{Z}_j$ is the interpolated value for grid node "j";
Z_i are the neighboring points;
d_{ij} is the distance between the grid node "j" and the neighboring point "i";
β is the weighting power (the Power parameter); and
δ is the Smoothing parameter.

One of the characteristics of Inverse Distance to a Power is the generation of "bull's-eyes" surrounding the position of observations within the gridded area. This effect can be reduced by smoothing the interpolation (SURFER manual).

Kriging

The first step in ordinary Kriging is to construct a variogram from the scatter point set to be interpolated. The variogram is a function that describes the spatial variation of the terrain. Once the model variogram is constructed, it is used to compute the weights used in Kriging. The variogram can be defined as half the expected squared difference between paired random functions (RFs) separated by the distance and direction vector (where anisotropy is considered) (Lloyd and Atkinson 2001). The basic equation used in ordinary Kriging is as follows (Kim S. et al 1999):

$$\hat{Z}_j = \sum_{i=1}^{n} w_i Z_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where:

n is the number of scattered points of the set;
w_i are weights assigned to each scatter point
$\hat{Z}_j$ is the interpolated value for grid node "j";
Z_i are the values of the neighboring points.

This equation is essentially the same as the equation used for inverse distance weighted interpolation (equation 1) except that rather than using weights based on an arbitrary function of distance, the weights used in Kriging are based on the variogram. Full details of ordinary Kriging can be found in (www.ems-i.com/gmshelp/Interpolation/Interpolation_Schems/Kriging/Ordinary_Kriging.htm).

Minimum curvature

The interpolated surface generated by Minimum Curvature is analogous to a thin, linearly elastic plate passing through each of the data values with a minimum amount of bending.

Thin-plate splines create a surface, which passes through control points and has the least possible change in slope at all points (Franke 1982) (geolibrary.uidaho.edu/courses/geog475/Lectures/5/). In other words, thin-plate splines fit the control points with a minimum-curvature surface. The approximation of thin-plate splines is of the form:

$$\hat{Z}(x,y) = \sum A_i d_i^2 \log d_i + a + bx + cy \qquad \ldots\ldots\ldots\ldots\ldots\ldots.... \qquad (4)$$

where:

x and y the X, Y coordinates of the point to be interpolated;

d_i the distance between interpolated and scatter points, $d_i^2 = (x - x_i)^2 + (y - y_i)^2$

$\hat{Z}$ is the interpolated value for grid node.

Thin-plate splines consist of two components:

$(a + bx + cy)$ represents the local trend function, which has the same form as a linear trend surface, and

$d_i^2 log d_i$ represents a basis function, which is designed to obtain minimum curvature surfaces.

The coefficients Ai, and a, b, and c are determined by a linear system of equations.

Thus, there are four steps to generate the final grid using the minimum curvature method. First, the least squares regression model is fit to the data. Second, the values of the planar regression model at the data locations are subtracted from the data values; this yields a set of residual data values. Third, the minimum curvature algorithm is used to interpolate the residuals at the grid nodes. Fourth, the values of the planar regression model at the grid nodes are added to the interpolated residuals, yielding a final interpolated surface (SURFER manual).

Modified Shepard's method

The simplest form of inverse distance weighted interpolation is sometimes called "Shepard's method". The equation used is as follows as presented in (Du 1996, Algarni and Elhassan 2001 , Lazzaro and Montefusco 2002 and Shen et al 1993):

$$\hat{Z}_j = \sum_{i=1}^{n} r_i^{-p} Z_i \Big/ \sum_{i=1}^{n} r_i^{-p} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (5)$$

where:

n is the number of scattered points of the set;
r_i denotes the Euclidean distance between the known and interpolated points;
$\hat{Z}_j$ is the interpolated value for grid node "j";
Z_i are the prescribed function values at the scatter points (e.g. the data set values).

This scheme has the advantage of a small storage requirement and a full independence from the space dimension, but suffers from low reproduction quality and high computational cost. Its modified version, called modified quadratic Shepard's method, presents improved reproduction quality and reduced complexity. Specifically, the interpolation function (Lazzaro and Montefusco 2002) is defined as:

$$F(x) = \sum_{i=1}^{n} w_i(x) Q_i(x) \qquad \text{..................................} \qquad (6)$$

where:

$Q_i(x)$ called *nodal function*, is a multivariate quadratic function that satisfies: $Q_i(x_i)=Z_i$;
n is the number of scattered points of the set;
$w_i(x)$ are weight functions.

The weight functions $w_i(x)$ are defined as:

$$w_i = \frac{\left[\dfrac{R-h_i}{Rh_i}\right]^2}{\sum_{j=1}^{n}\left[\dfrac{R-h_j}{Rh_j}\right]^2} \qquad \text{..} \qquad (7)$$

where:

h_i is is the distance from the interpolation point to scatter point "i";
R is a radius of influence about the scattered point;
n is the total number of scatter points;

for

$$R - r_i = \begin{cases} R - r_i & \text{if } r_i < R \\ 0 & \text{if } r_i \geq R \end{cases} \qquad \text{..........................} \qquad (8)$$

The weights $w_i(x)$ satisfy the cardinality relations and constitute a partition of unity, namely:

$$\sum_{i=1}^{n} w_i(x) = 1 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (9)$$

Moreover the weight functions have continuous partial derivatives which are zero at the interpolation points, so the interpolating function *F*, at the data points, maintains the local shape properties of the nodal functions (Lazzaro and Montefusco 2002).

The weight function is a function of Euclidean distance and is radially symmetric about each scatter point. As a result, the interpolating surface is somewhat symmetric about each point and tends toward the mean value of the scatter points between the scatter points. Shepard's method has been used extensively because of its simplicity.

Natural neighbour

The basic equation used in natural neighbor interpolation is identical to the one used in Inverse Distance Weighted (IDW) interpolation (Boissonnat and Cazals 2002):

$$F(x, y) = \sum_{i=1}^{n} w_i\, Q_i(x, y) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (10)$$

The difference between IDW interpolation and natural neighbor interpolation is the method used to compute the weights and the method used to select the subset of scatter points used for interpolation.

Weights used in natural neighbor interpolation are based on the concept of local coordinates. Local coordinates define the "neighborliness" or amount of influence any scatter point will have on the computed value at the interpolation point. This neighborliness is entirely dependent on the area of influence of the Thiessen polygons of the surrounding scatter points.

To define the local coordinates for the interpolation point, P_n, the area of all Thiessen polygons in the network must be known. Temporarily inserting P_n into the triangulated irregular network (TIN) causes the TIN and the corresponding Thiessen network to change, resulting in new Thiessen areas for the polygons in the neighborhood of P_n.

The concept of local coordinates is shown graphically in the following figure. Points 1-10 are scatter points and P_n is a point where some value associated with points 1-10 is to be interpolated. The dashed lines show the edges of the Thiessen network before P_n is temporarily inserted into the TIN and the solid lines show the edges of the Thiessen network after P_n is inserted.

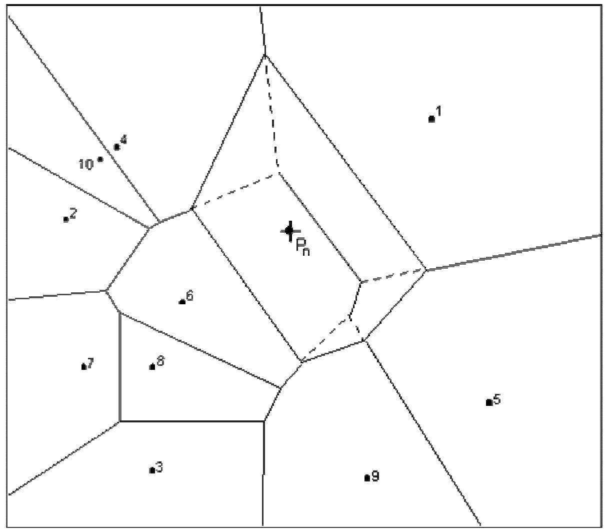

Fig. (1) Overlapping Thiessen Polygon Areas Used in Computation of weight.

Only those scatter points whose Thiessen polygons have been altered by the temporary insertion of P_n are included in the subset of scatter points used to interpolate a value at P_n. In this case, only points 1, 4, 5, 6, & 9 are used. The local coordinate for each of these points with respect to P_n is defined as the area shared by the Thiessen polygon defined by point P_n and the Thiessen polygon defined by each point before point P_n is added. The greater the common area, the larger the resulting local coordinate, and the larger the influence or weight the scatter point has on the interpolated value at P_n.

If we define k(n) as the Thiessen polygon area of P_n and $k_m(n)$ as the difference in the Thiessen polygon area of a neighboring scatter point, P_m, before and after P_n is inserted, then the local coordinate $\lambda_m(n)$ is defined as:

$$\lambda_m(n) = \frac{k_m(n)}{k(n)} \qquad \text{.................................} \qquad (11)$$

The local coordinate $\lambda_m(n)$ varies between zero and unity and is directly used as the weight, $w_m(n)$, in the interpolation equation. If P_n is at precisely the same location as Pm, then the Thiessen polygon areas for P_n and Pm are identical and $\lambda_m(n)$ has a value of unity. In general, the greater the relative distance Pm is from P_n, the smaller its influence on the final interpolated value.

Nearest neighbour

The following describes perhaps the simplest method of "smoothly" approximating height values on a surface given a collection of randomly distributed samples. It is often used to derive estimates of the surface height at the vertices of a regular grid from irregularly spaced samples (Bourke 1998).

Consider n height samples, that is, we have n triples (x_i, y_i, z_i). We want to estimate the height Z given a position on the plane (x, y). The general form of the so called "nearest neighbor weighted interpolation" also for estimating z is given by the following (Bourke 1998):

$$\hat{Z} = \begin{cases} \dfrac{\displaystyle\sum_{i=0}^{n-1} \frac{Z_i}{\left[(x_i - x)^2 + (y_i - y)^2\right]^{p/2}}}{\displaystyle\sum_{i=0}^{n-1} \frac{1}{\left[(x_i - x)^2 + (y_i - y)^2\right]^{p/2}}} & x_i \neq x \;\; or \;\; y_i \neq y \\ Z_i & x_i = x \;\; and \;\; y_i = y \end{cases} \quad \ldots\ldots\ldots \quad (12)$$

Where p generally determines relative importance of distant samples. Note the denominator above gives a measure of how close the point being estimated is from the samples. Naturally if a sample is close then it has a greater influence on the estimate than if the sample is distant.

Polynomial regression

A well-known global method in modeling a continuous surface is trend surface analysis. The analysis uses a polynomial equation to approximate points with known values. The equation (the trend surface model) can then be used to estimate values at other points. A third-order polynomial uses the equation:

$$\hat{Z}(x,y) = b_0 + b_1 x + b_2 y + b_3 x^2 + b_4 xy + b_5 y^2 + b_6 x^3 + b_7 x^2 y + b_8 xy^2 + b_9 y^3 \quad \ldots\ldots(13)$$

where the attribute value Z is a function of X and Y. Similar to multiple regression analysis, the b coefficients can be estimated from the control points. After the b coefficients are derived, the equation can be used to estimate Z values at other points.

Radial basis function

Radial Basis Function interpolation is a diverse group of data interpolation methods. In terms of the ability to fit your data and to produce a smooth surface, the multiquadric method is considered by many to be the best. All of the Radial Basis Function methods are exact interpolators, so they attempt to honor your data. You can introduce a smoothing factor to all the methods in an attempt to produce a smoother surface.
The Radial Basis function can be defined as following (Lazzaro and Montefusco 2002, Driscoll and Fornberg 2002):

We define two quantities which, in some sense, measure the density of the data set *X*.

The first is the *separation distance*

$$q(X) = \min_{x_j \neq x_k} \|x_k - x_j\| / 2 \quad \text{..................................} \quad (14)$$

where $\| \, . \, \|$ denotes the Euclidean distance between two points.

namely, half the distance between the closest pair of points in *X*.

The second is the *fill distance*, which gives the radius of the largest inner empty sphere,

$$h(X) = \max \min_{1 \le j \le n} \|x - x_j\| \quad \text{..........................} \quad (15)$$

Interpolation surface passes through all the available data points.

$$F(x) = \sum_{i=1}^{n} w_i \, \varphi(\|x - x_i\|) \quad \text{..................................} \quad (16)$$

where $\| \, . \, \|$ denotes the Euclidean norm, and the *wi's* are linear weights.

The known data points are the centers of the radial-basis functions.
Define

$$\Phi = \{\|\varphi(x_j - x_i\|\} \quad j, i = 1, 2, \ldots\ldots, n \quad \text{..........................} \quad (17)$$

N x N matrix

and the

N x 1 weight vector

$$w = \{w_i\}_{i=1}^{n} \quad \text{..} \quad (18)$$

***d*=**{ *di* }, *i*= 1, 2, … *N*

= desired response vector

Φ ***w*= *d*** Interpolation equation.

Provided that Φ is nonsingular,

$$w = \Phi^{-1} d \quad \text{...} \quad (19)$$

where Φ^{-1} = inverse of interpolation matrix Φ.

Hence, given the data {(*xi, di*)} Eq. (19) defines the solution to the interpolation problem.

Triangulation with linear interpolation.

The Triangulation with Linear Interpolation method uses the optimal Delaunay triangulation. Delaunay triangulation has several advantages over other triangulation methods: The triangulation is independent of the order the points are processed, The triangles are as equi-angular as possible, thus reducing potential numerical precision problems created by long skinny triangles and Ensures that any point on the surface is as close as possible to a node.
A surface value is obtained by intersecting a vertical line with the plane defined by the three nodes of the triangle. The generalized equation for LINEAR interpolation of a point (x, y, z) in a triangle facet is:

$$Ax + By + Cz + D = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (20)$$

where:
A, B, C, and D are constants determined by the coordinates of the triangle's three nodes.

Each triangle defines a plane over the grid nodes lying within the triangle, with the tilt and elevation of the triangle determined by the three original data points defining the triangle. All grid nodes within a given triangle are defined by the triangular surface. Because the original data are used to define the triangles, the data are honored very closely.
Triangulation with Linear Interpolation works best when your data are evenly distributed over the grid area. Data sets that contain sparse areas result in distinct triangular facets on the map.

THE STUDY AREA AND MATERIALS

A part (200 x 200 m) of a digital contour map of the mountains around Mina's valley in Makka was used in this study. The elevations are from 460 m to 580 m and the contour interval is 10 m. This part was chosen because it has different shapes of the contour lines and different slopes 20%, 50% and 70% in different directions.
On screen digitizing technique was used to get the coordinates of the scattered points along the contour lines once and another time to get the coordinates of a regular grid of 10 m spacing by interpolation, to test the effect of the data pattern on the DEM accuracy. The borders of the digitized map were in the north-south and east-west directions.
The two data sets (scattered and girded) were rotated by 15, 30, 45, 60, 75, and 90 degree from north to test the effect of the orientation of the data on the accuracy. This makes 14 data sets interpolated using 9 interpolation methods which produce 126 models.
The interpolation was done using SURFER software which offers the nine interpolation methods. Although a computer programs for the interpolation methods

can be built. The author preferred to use the available software to make the paper valuable for the software users.

ANALYSIS OF THE RESULTS

Visualization provides a powerful mechanism for identifying spatial distributions and DEM uncertainty (Wood and Fisher, 1993). Contours still traditional visualization technique. The visual comparison between the original and resultant maps was used to check the accuracy of the DEM. The results can be analyzed as following:

Gridded data

Effect of interpolation method

The results of the gridded data oriented to north are shown in figure (2) with the original contour map. From the figure it is clear that all interpolation methods gave the same results except the polynomial regression which gave the worst result, it is not real interpolator as mentioned by (Fehmi 2001). The natural neighboring reduced the size of the map by twice the grid spacing from each side.

The orientation (tilting from north) has no effect on the Inverse distance to a power, Kriging, Modified Shepard's method, Natural neighbour, Radial basis function and Triangulation with linear interpolation. It changes the contour shape for the maps produced by using Minimum curvature, Nearest neighbour and Polynomial regression. Figure (3) show the results of 30 degree oriented gridded data with all interpolation methods. The minimum curvature and polynomial methods make an extrapolation to cover a new area one of its corners has the minimum coordinates and the opposite one has the maximum coordinates of the data set. If a frame represents the borders of the actual area was superimposed on the resultant contour map produced by minimum curvature method, better result can be obtained as shown in figure (3). The nearest neighbour method produced corrugated contour lines which destroy the shape of the contour map. Many trails were made to enhance the shape of the contour lines by using different search radius during gridding but the same result was obtained. This means that the orientation is the responsible for this result.

The natural neighboring method still reduces the size of the map by twice the grid spacing from each side.

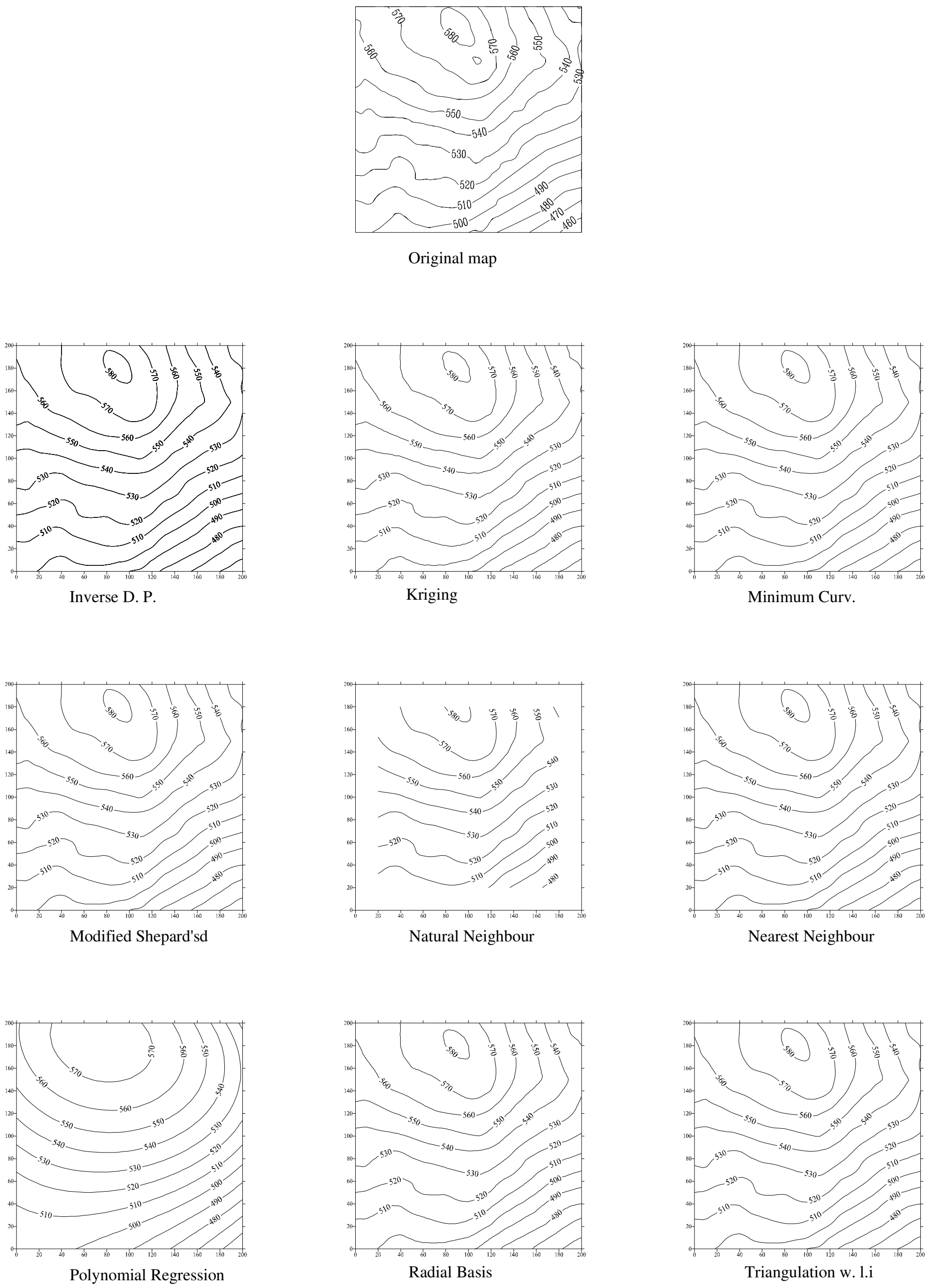

Fig. (2) The results of the gridded data oriented to north

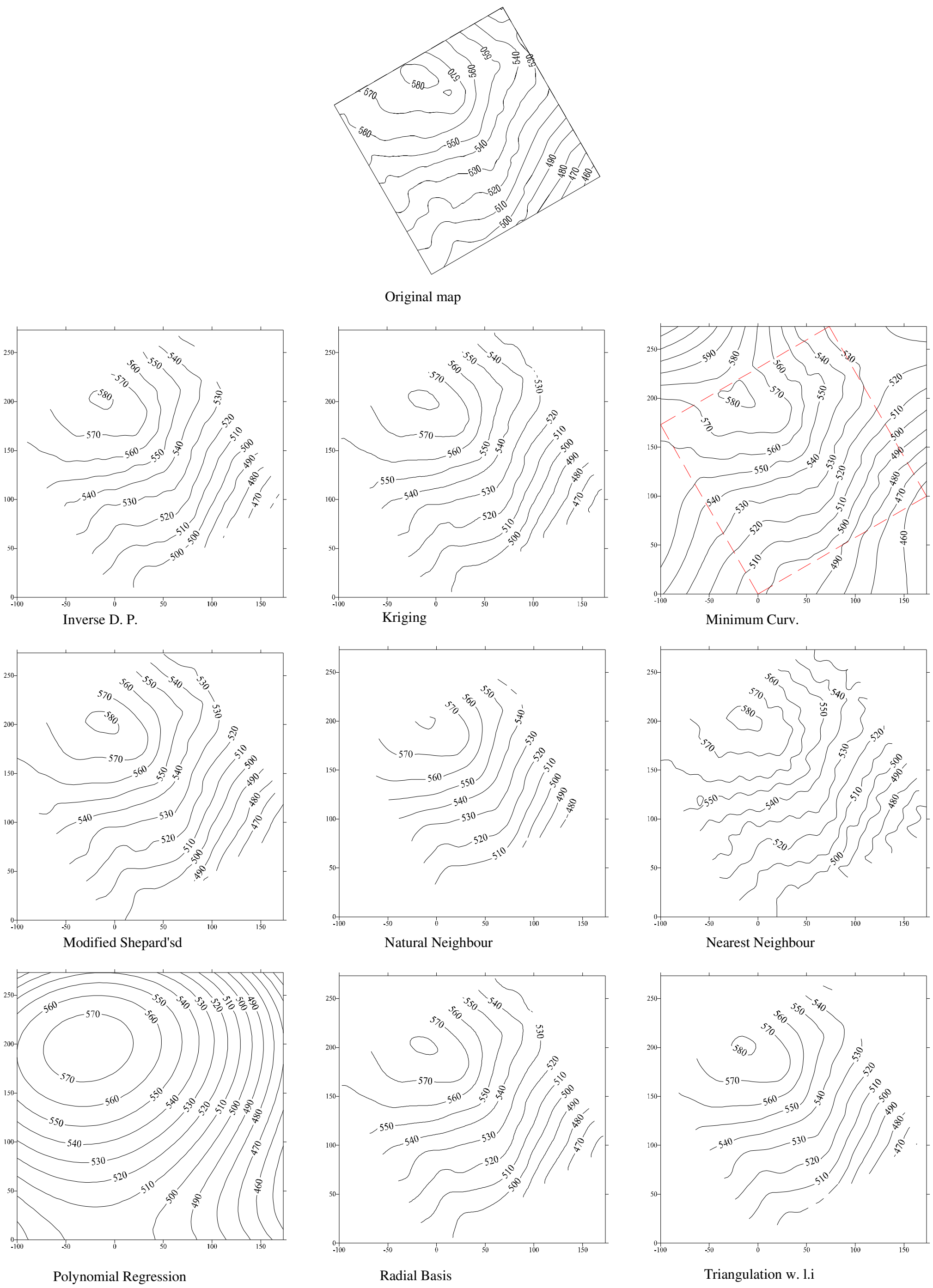

Fig. (3) The results of the gridded data tilted by 30 degree from north

Effect of orientation

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The minimum curvature method, nearest neighbor and the polynomial regression are affected by the orientation as mentioned above. The results of the minimum curvature method and nearest neighbor are shown in figures (4) and (5) respectively. From the figure one can notice that the results of the gridded data rotated by 90 degree (oriented to east) are exactly the same as that oriented to north. The results of the data tilted by 45 degrees are the closest to that of 0 and 90 degrees, this tilting angle act as a center, e.g. the results around it is nearly similar. The results of gridded data tilted by 30 degrees are nearly the same as that tilted by 60 degrees and the results of gridded data tilted by 15 degrees are the same as that tilted by 75 degrees. For all tilting angles a frame of the actual size can help to get red of extrapolation and to get a result close to the correct shape.

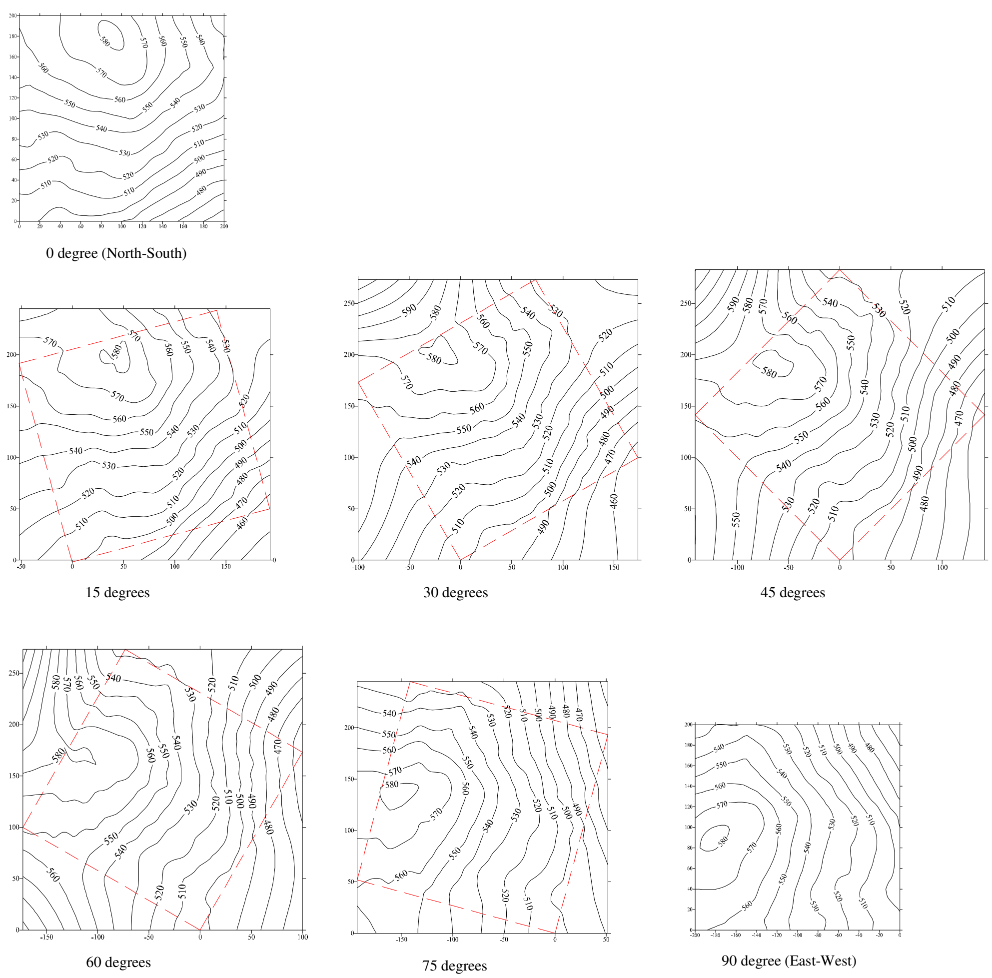

Fig. (4) The results of the gridded data interpolated by using Minimum curvature method

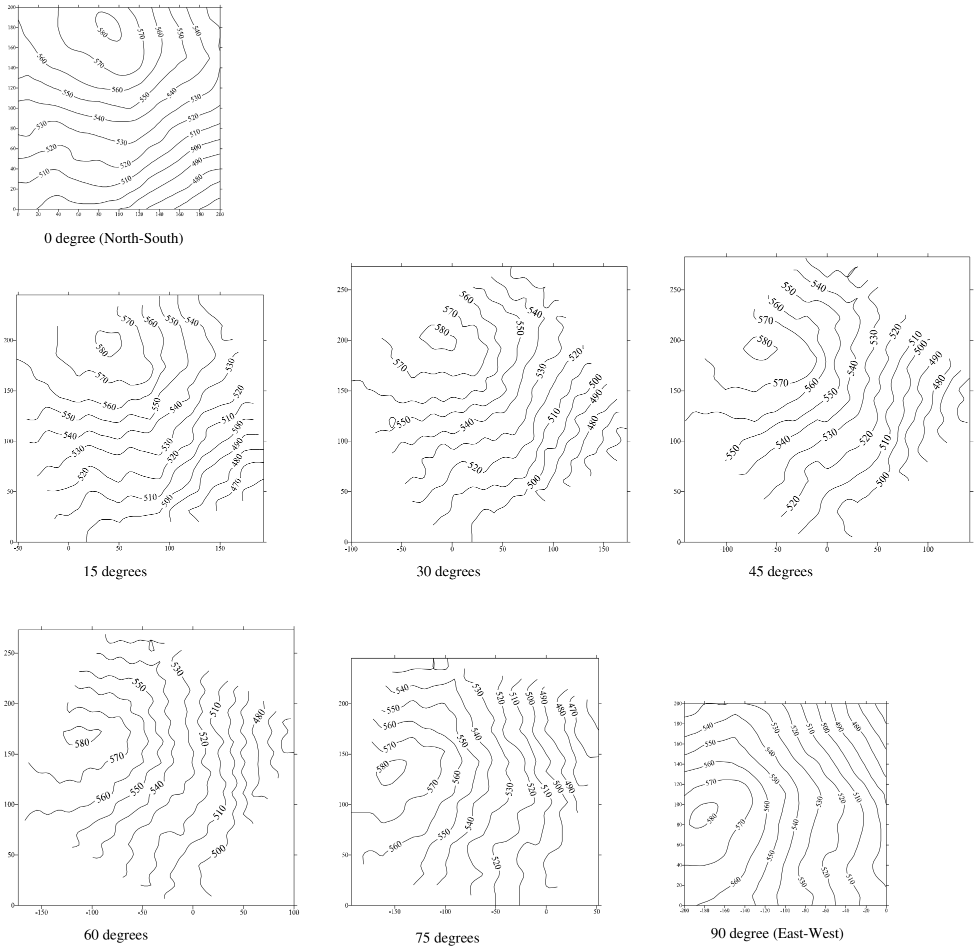

Fig. (5) The results of the gridded data interpolated by using nearest neighbor method

Scattered data

Effect of interpolation method

The results of the scattered data oriented to north are shown in figure (6) with the original contour map. From the figure it is clear that the modified Shepard's method, radial basis function and Kriging interpolation are the best methods. The results of minimum curvature, inverse distance to power and triangulation with linear interpolation gave results close to the correct. The results of inverse distance to power and triangulation with linear interpolation are nearly the same. The polynomial regression and nearest neighbor methods gave the worst results. The natural neighboring reduced the size of the map by twice the spacing of the generated grid from each side.

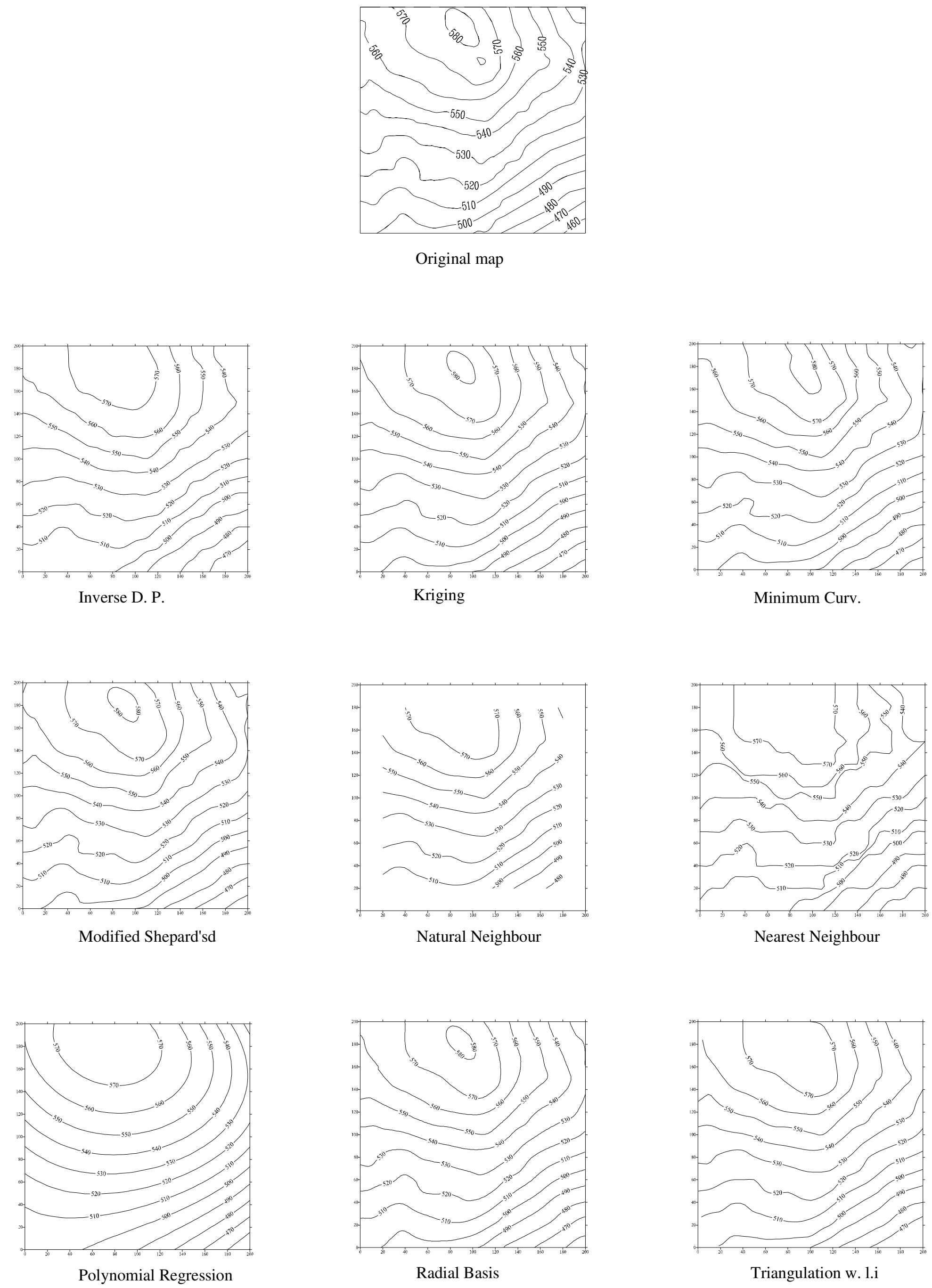

Fig. (6) The results of the scattered data oriented to north

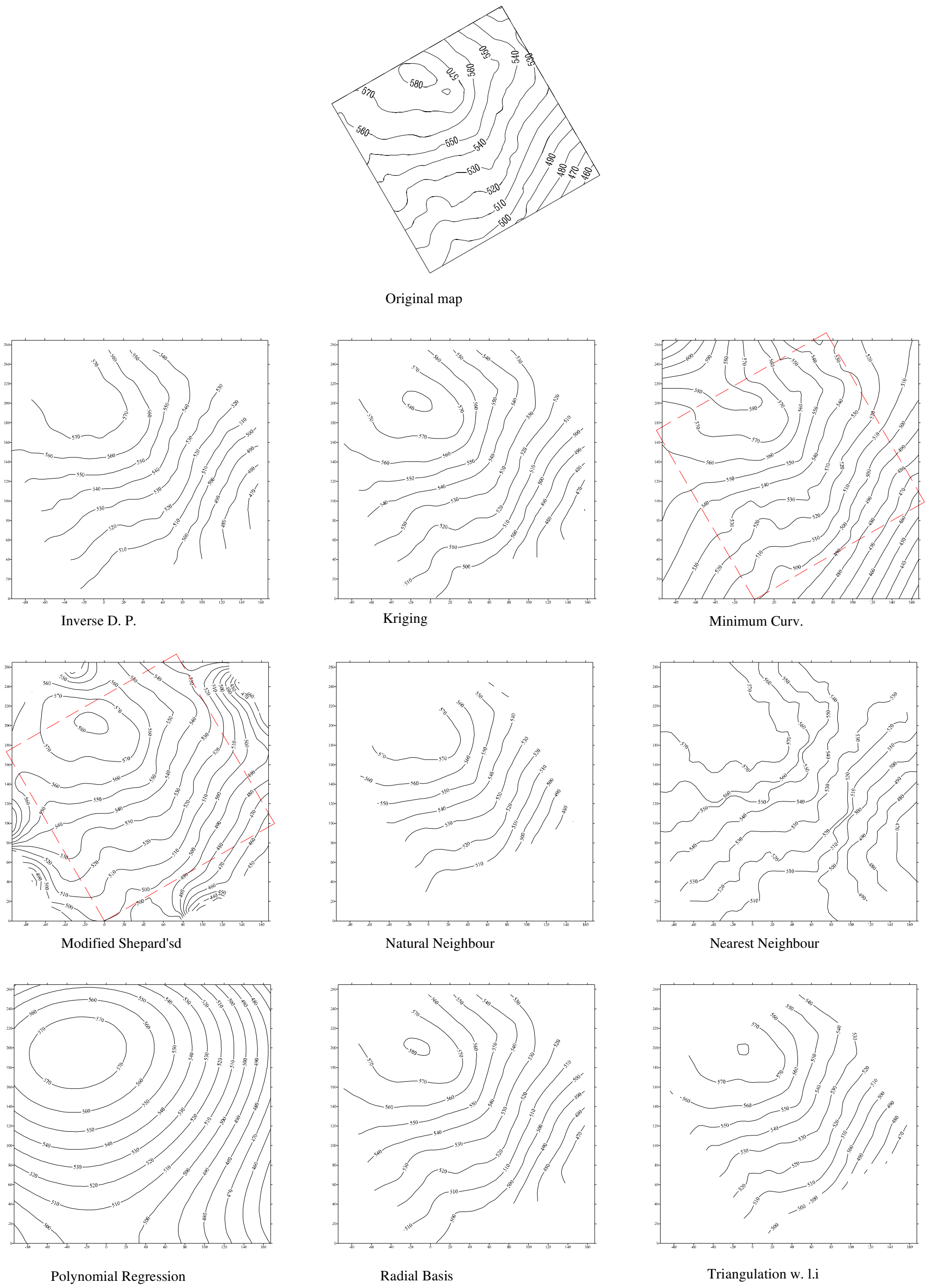

Fig. (7) The results of the scattered data tilted by 30 degree from north

The orientation has no effect on the Inverse distance to a power, Kriging, Natural neighbour, radial basis function and Triangulation with linear interpolation. It changes the contour shape for the maps produced by using Minimum curvature, Modified Shepard's method, nearest neighbour and Polynomial regression. Figure (7) show the results of 30 degree oriented scattered data with all interpolation methods. The minimum curvature, modified Shepard's method and polynomial methods make extrapolation to cover a new area as mentioned with the gridded data. If a frame represents the borders of the actual area was superimposed on the resultant contour map produced by minimum curvature method, better result can be obtained as shown in figure (7) especially with the modified Shepard's method. The nearest neighbour method gave the same result as what it gave with the gridded data beside the extrapolation. The natural neighboring method still reduces the size of the map by twice the spacing of the generated grid from each side.

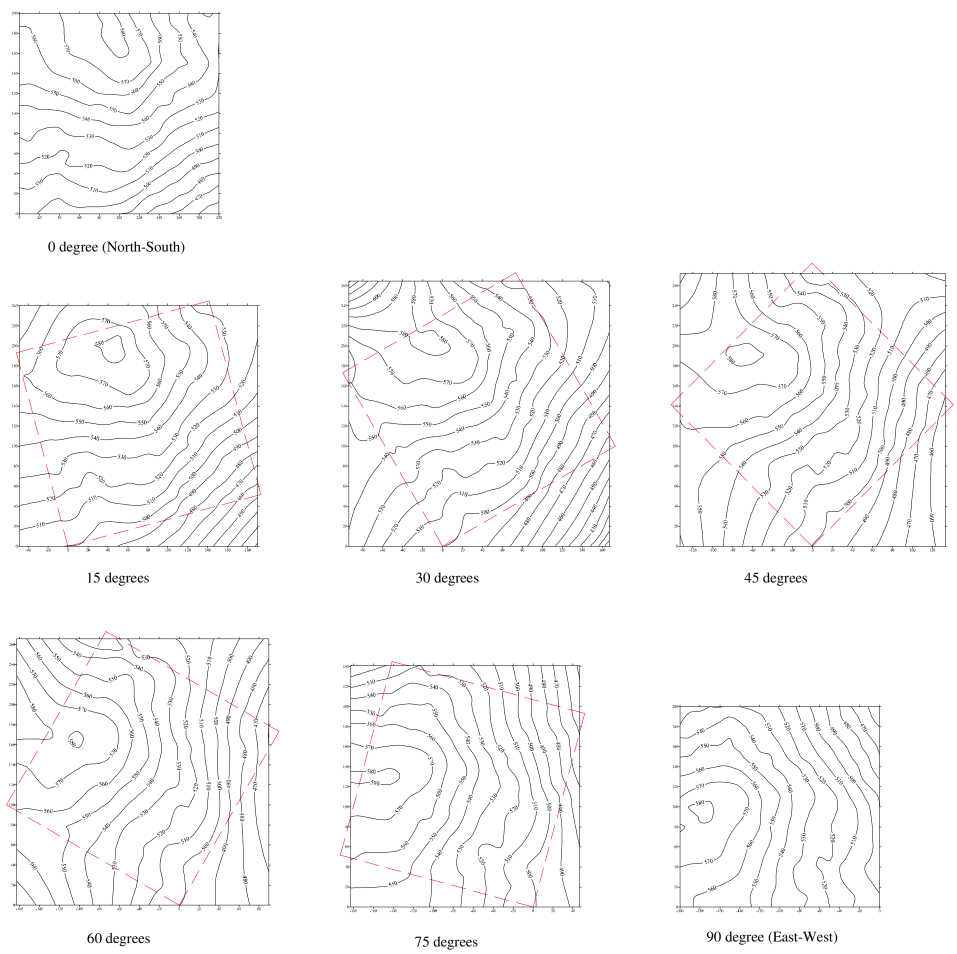

Fig. (8) The results of the gridded data interpolated by using nearest neighbor method

Effect of orientation

The minimum curvature method, modified Shepard's method, nearest neighbor and the polynomial regression are affected by the orientation as mentioned above. The results of the minimum curvature method and modified Shepard's method are shown in figures (8) and (9) respectively. From the figure one can notice that the results of the scattered data rotated by 90 degree are the same as that oriented to north. For all tilting angles a frame of the actual size can help to get red of extrapolation and to get a result close to the correct shape. The polynomial regression and nearest neighbor gave the worst results and it is recommended not to use them with scattered data pattern.

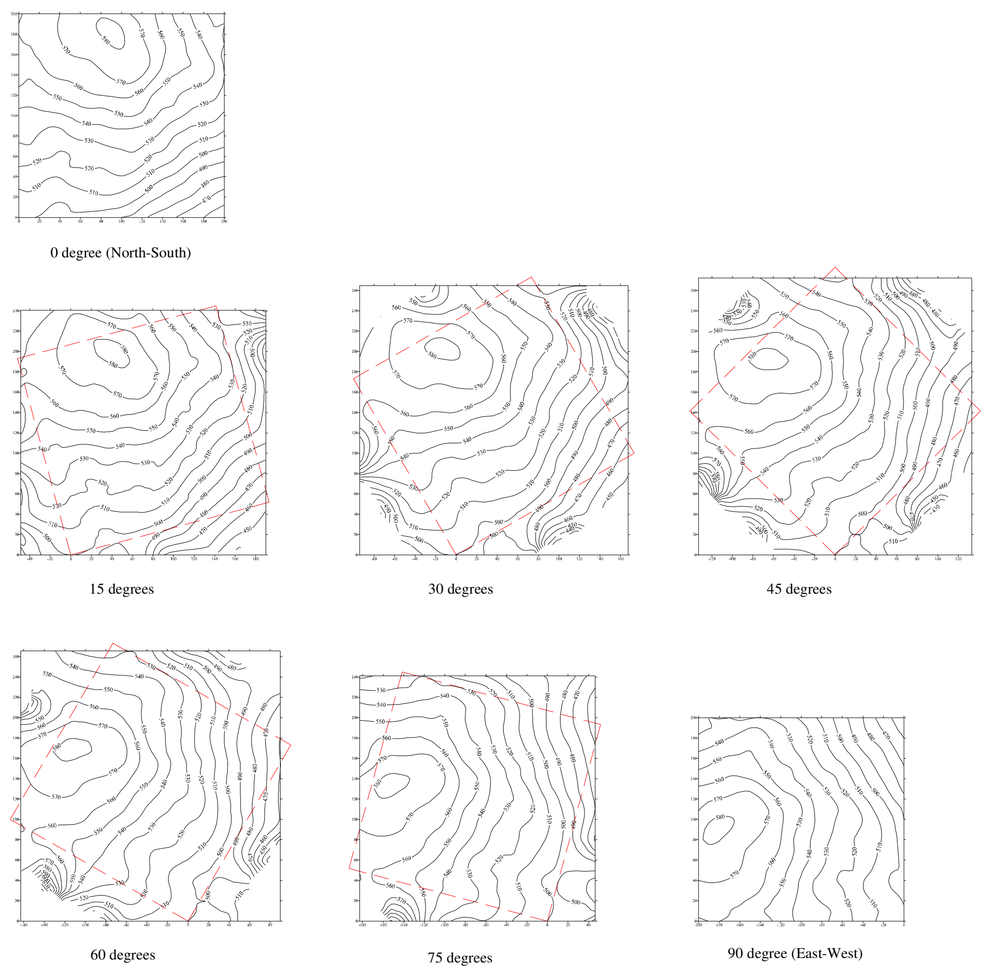

Fig. (9) The results of the gridded data interpolated by using modified Shepard's method

CONCLUSIONS

From the discussions the following remarks can be concluded:

1- The optimum interpolation methods for all data patterns and orientations are Kriging and radial basis function.
2- Inverse distance to power and triangulation with linear interpolation gave the same results. Their results are close to that of the optimum methods.
3- The shortcoming of minimum curvature and modified Shepard's methods is the extrapolation especially when using with scattered data, better results can be obtained when a frame of the actual map size is used to get ret of the extrapolation.
4- Nearest neighbor method is suitable only for gridded data oriented to north or east. It is not recommended to be used for contouring.
5- Natural neighbor method gave results near to that of inverse distance to power but it reduces the map dimension by twice the grid (or generated grid) spacing from each side.
6- Polynomial regression gave the worst results and it is not recommended to be used for contouring.
7- The optimum orientation for the data sets (gridded or scattered points) is to north-south or to east-west.
8- It is recommended to rotate the tilted data to be oriented towards north before contour generation and re-rotate the map to the former direction.
9- The optimum search radius was checked during the research and it was found to be just greater than the grid spacing (for gridded data), and just greater than the maximum distance between points (for scattered data).

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