DIRECT NUMERICAL METHOD FOR CALCULATING THE PARAMETERS OF HORIZONTAL CIRCULAR CURVES

Ragab KHALIL
Lecturer at Civil Engineering Department,
Faculty of Engineering,
Assiut University,
Assiut, Egypt.

ملخص:

للمنحنى الدائري الأفقي استخدامات عديدة في مجال الهندسة المدنية كإنشاء الطرق والسكك الحديدية والمجارى المائية، والمنحنى الدائري الافقى يمكن تعريفه بسبعة عناصر رئيسية هي (١) نصف قطر المنحنى، (٢) زاوية تقاطع المماسان، (٣) طول المماس، (٤) السهم الخارجي، (٥) سهم المنحنى، (٦) الوتر الكلى، (٧) طول المنحنى. وفى الحالة العادية لحل المنحنى يمكن حساب عناصر المنحنى بطريقة مباشرة بمعلومية نصف القطر وزاوية تقاطع المماسان، وفى بعض التطبيقات العملية يكون هذان العنصران مجهولين وتستخدم عدة طرق مثل Newton- Raphson method and iteration methods مبنية كلها على التكرار لإيجاد نصف القطر وزاوية تقاطع المماسين بمعلومية أى عنصرين آخرين من عناصر المنحنى.

وهذا البحث يعرض طريقة مباشرة لإيجاد نصف قطر المنحنى وزاوية التقاطع بين المماسين كما يعرض ثلاث حالات جديدة لم تتعرض لها الأبحاث السابقة في هذا الموضوع وقد تم تطبيق الطريقة المقترحة على مثال حسابي وأعطت نفس النتائج التي أعطتها الطرق الأخرى.

ABSTRACT

The horizontal circular curve can be described by seven elements: (1) Radius of the curve; (2)deflection angle between tangents; (3) tangent distance; (4) external distance; (5) middle ordinate; (6) long chord; and (7) length of the curve. When the radius and deflection angle are given, the other five curve elements can be directly computed. In some practical problems, the radius and the deflection angle are unknown; two other elements must be known to solve the problem. Seven cases must be solved depending on the known curve elements, as mentioned by other authors. This paper present three other cases and a direct method is proposed for two cases of the earlier seven.

KEY WORDS: Circular curve; Layout; Direct method

INTRODUCTION

The horizontal circular curve can be defined as shown in Fig. (1) by seven elements: (1) Radius of the curve, R; (2) deflection angle between tangents, I; (3)

tangent distance, T; (4) external distance, E; (5) middle ordinate, M; (6) long chord, Lc; and (7) length of the curve, L. PC is the point of curvature, PI is the point of intersection and PT is the point of tangent. When the radius and deflection angle are given, the other five curve elements can be directly computed as presented in (Anderson et al. 1985; Moffit and Bouchard 1987; Wolf and brinker 1989).

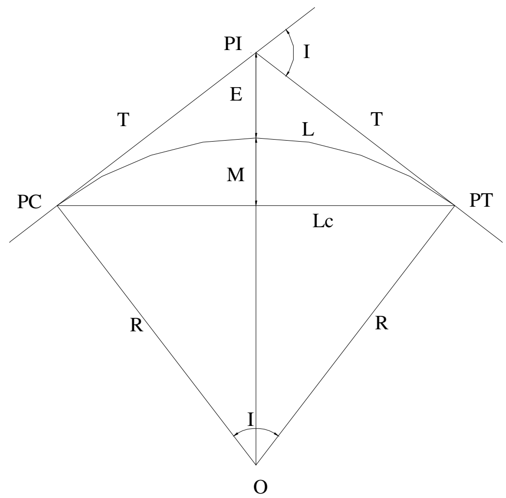

Fig. (1) Principal elements of Horizontal Circular Curve

Seven cases in which the curve radius and the deflection angle are unknown have been solved by Chen and Hwang (1992), Li (1993) and Easa (1994). The various cases they addressed are shown in table (1).

The solution of case (1) is direct, but the other six cases must be iteratively solved because R and I cannot be explicitly expressed in terms of the known elements as stated by Easa (1994).

Table (1) The studied cases of the Circular Curve

Case number (1)	Unknown elements (2)	Given elements (3)	Formulas (4)
1	*R, I*	*T, E*	$\boldsymbol{T}^2 + \boldsymbol{R}^2 = (\boldsymbol{R}+\boldsymbol{E})^2, \quad \sin(\boldsymbol{I}/2) = \boldsymbol{T}/(\boldsymbol{R}+\boldsymbol{E})$
2	*R, I*	*T, L*	$\boldsymbol{I} = 180^\circ \boldsymbol{L}/\pi\boldsymbol{R}, \quad \tan(\boldsymbol{I}/2) = \boldsymbol{T}/\boldsymbol{R}$
3	*R, I*	*L, E*	$\boldsymbol{I} = 180^\circ \boldsymbol{L}/\pi\boldsymbol{R}, \quad \cos(\boldsymbol{I}/2) = \boldsymbol{R}/(\boldsymbol{R}+\boldsymbol{E})$
4	*R, I*	*L, M*	$\boldsymbol{I} = 180^\circ \boldsymbol{L}/\pi\boldsymbol{R}, \quad \boldsymbol{M} = \boldsymbol{R}[1-\cos(/2)]$
5	*R, I*	*L, Lc*	$\boldsymbol{I} = 180^\circ \boldsymbol{L}/\pi\boldsymbol{R}, \quad \boldsymbol{Lc} = 2\boldsymbol{R}\sin(\boldsymbol{I}/2)$
6	*R, I*	*T, M*	$\tan(\boldsymbol{I}/2) = \boldsymbol{T}/\boldsymbol{R}, \quad \boldsymbol{M} = \boldsymbol{R}[1-\cos(/2)]$
7	*R, I*	*E, Lc*	$\cos(\boldsymbol{I}/2) = \boldsymbol{R}/(\boldsymbol{R}+\boldsymbol{E}), \quad \sin(\boldsymbol{I}/2) = \boldsymbol{Lc}/2\boldsymbol{R}$

Chen and Hwang (1992) proposed using Newton-Raphson (NR) method for solving these cases. Their method had some short comes as stated by Easa (1993) that it needs an initial guessed value of the curve radius. It requires computing the derivatives and may not converge for some initial values.

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Easa (1994) suggested a numerical method, called the iteration method, for finding the solution of the circular curve problems in which the curve parameters cannot be determined directly. He expressed the iteration function in terms of the radius of the circular curve *R*. The iteration method proposed by Easa is simpler than the Newton-Raphson (NR) method proposed by Chen and Hwang because it requires no derivatives and it usually converges for a larger set of initial values (Dubeau 1995). Easa (1994) concluded that there are two solutions of case (6), his method similar to NR method provides one solution and the other solution must be found using exhaustive search.
Li (1993) also proposed a numerical iteration method, for solving the parameters of the horizontal circular curve. He expressed the iteration function in terms of the deflection angle between tangents, *I* and derived a formula for the initial estimates of *I*. Li (1995) explain the compound iterative scheme which make the iterative process more efficient than the simple iterative scheme.
Easa (1995) said that similar to Chen and Hwang method, however, Li's method has not addressed the fact that case (6) has two solutions.
This paper presents three more cases never be mentioned before for solving the problem and proposed a direct solution for cases (6 and 7).

DIRECT METHOD

In this method a new direct formula was derived for case (6) and (7) and for a new three cases (8), (9) and (10). Also a simple form is derived for case (1). The formulas are shown in table (2). The derived relationship of case (6) is presented and the relationships for other cases are listed

Case (6)

$$\cos(\boldsymbol{I}/2)=\frac{\boldsymbol{R}-\boldsymbol{M}}{\boldsymbol{R}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\sin(\boldsymbol{I}/2)=\frac{\boldsymbol{T}(\boldsymbol{R}-\boldsymbol{M})}{\boldsymbol{R}^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Rise equation (1) and (2) to the power of 2 and adding them together we get

$$\frac{(\boldsymbol{R}-\boldsymbol{M})^2}{\boldsymbol{R}^2}+\frac{\boldsymbol{T}^2(\boldsymbol{R}-\boldsymbol{M})^2}{\boldsymbol{R}^4}=1 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

After the needed abbreviations the final formula can be written as

$$(2\boldsymbol{M})\boldsymbol{R}^3-(\boldsymbol{M}^2+\boldsymbol{T}^2)\boldsymbol{R}^2+(2\boldsymbol{T}^2\boldsymbol{M})\boldsymbol{R}-(\boldsymbol{T}^2\boldsymbol{M}^2)=0 \quad \ldots\ldots\ldots \quad (4)$$

The formulas for case (1), (8), (9) and (10) are simple while that for case (6) and (7) are in the form of the cubic polynomial.

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Table (2) The new formulas of the Circular Curve

Case number (1)	Unknown elements (2)	Given elements (3)	Formulas (4)
1	*R, I*	*T, E*	$\tan(\boldsymbol{I}/4)=\boldsymbol{E}/\boldsymbol{T}, \quad \boldsymbol{R}=\boldsymbol{T}/\tan(\boldsymbol{I}/2)$
6	*R, I*	*T, M*	$(2\boldsymbol{M})\boldsymbol{R}^3-(\boldsymbol{M}^2+\boldsymbol{T}^2)\boldsymbol{R}^2+(2\boldsymbol{T}^2\boldsymbol{M})\boldsymbol{R}-(\boldsymbol{T}^2\boldsymbol{M}^2) = 0,$ $\tan(\boldsymbol{I}/2)=\boldsymbol{T}/\boldsymbol{R}$
7	*R, I*	*E, Lc*	$(8\boldsymbol{E})\boldsymbol{R}^3+(4\boldsymbol{E}^2-\boldsymbol{Lc}^2)\boldsymbol{R}^2-(2\boldsymbol{E}\,\boldsymbol{Lc}^2)\boldsymbol{R}-(\boldsymbol{Lc}^2\boldsymbol{E}^2) = 0,$ $\cos(\boldsymbol{I}/2)=\boldsymbol{R}/(\boldsymbol{R}+\boldsymbol{E})$
8	*R, I*	*T, Lc*	$\cos(\boldsymbol{I}/2)=\boldsymbol{Lc}/(2\boldsymbol{T}), \quad \boldsymbol{R}=\boldsymbol{Lc}/(2\sin(\boldsymbol{I}/2))$
9	*R, I*	*E, M*	$\cos(\boldsymbol{I}/2)=\boldsymbol{M}/\boldsymbol{E}, \quad \boldsymbol{R}=(\boldsymbol{E}+\boldsymbol{M})/(\tan(\boldsymbol{I}/2)\sin(\boldsymbol{I}/2))$
10	*R, I*	*M, Lc*	$\boldsymbol{R}=\boldsymbol{Lc}^2/8\boldsymbol{M}+\boldsymbol{M}/2, \quad \sin(\boldsymbol{I}/2)=\boldsymbol{Lc}/2\boldsymbol{R}$

SOLVING THE CUBIC POLYNOMIAL

To solve the general cubic polynomial

$$\boldsymbol{ax}^3+\boldsymbol{bx}^2+\boldsymbol{cx}+\boldsymbol{d}=0, \qquad \boldsymbol{a}\neq 0 \qquad \ldots\ldots\ldots\ldots \qquad (5)$$

First let

$$\boldsymbol{p}=\frac{3\boldsymbol{ac}-\boldsymbol{b}^2}{9\boldsymbol{a}^2} \qquad \ldots\ldots\ldots\ldots \qquad (6)$$

$$\boldsymbol{q}=\frac{9\boldsymbol{abc}-2\boldsymbol{b}^3-27\boldsymbol{a}^2\boldsymbol{d}}{54\boldsymbol{a}^3} \qquad \ldots\ldots\ldots\ldots \qquad (7)$$

$$\boldsymbol{D}=\boldsymbol{p}^3+\boldsymbol{q}^2 \qquad \ldots\ldots\ldots\ldots \qquad (8)$$

Suppose p and q are real numbers.

$D > 0$: one real and two complex conjugate solutions
$D = 0$: three real solutions, at least two of which are equal
$D < 0$: three distinct real solutions

Now let

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$\alpha = q + \sqrt{D}$ and .... (9)

$\beta = q - \sqrt{D}$ .... (10)

The solutions are given by

$$\sqrt[3]{\alpha} + \sqrt[3]{\beta} - \frac{b}{3a} \quad \text{.....................................} \quad (11)$$

$$-\frac{1}{2}\left(\sqrt[3]{\alpha} + \sqrt[3]{\beta}\right) - \frac{b}{3a} + \frac{\sqrt{3}}{2}\left(\sqrt[3]{\alpha} - \sqrt[3]{\beta}\right) i \quad \text{........................} \quad (12)$$

$$-\frac{1}{2}\left(\sqrt[3]{\alpha} + \sqrt[3]{\beta}\right) - \frac{b}{3a} - \frac{\sqrt{3}}{2}\left(\sqrt[3]{\alpha} - \sqrt[3]{\beta}\right) i \quad \text{........................} \quad (13)$$

If $D < 0$, it is easier to let

$$\theta = \cos^{-1}\left(\frac{q}{\sqrt{-p^3}}\right) \quad \text{..} \quad (14)$$

and obtain the solutions from the following formulas

$$2\sqrt{-p}\cos\left(\frac{\theta}{3}\right) - \frac{b}{3a} \quad \text{..} \quad (15)$$

$$2\sqrt{-p}\cos\left(\frac{\theta}{3} + \frac{2\pi}{3}\right) - \frac{b}{3a} \quad \text{.....................................} \quad (16)$$

$$2\sqrt{-p}\cos\left(\frac{\theta}{3} + \frac{4\pi}{3}\right) - \frac{b}{3a} \quad \text{.....................................} \quad (17)$$

APPLICATION

The proposed method was applied to the numerical example of Chen and Hwang (1992). The given elements for various cases are: *L = 467.310 m; T = 273.935 m; E = 92.990 m; M = 73.773 m; and Lc = 434.655 m*. The results are shown in table (3). The results of *R* and *I* are the same for all cases except case (6). The proposed method showed that there are three solutions for case (6). The first solution is *R = 356.995 m* and *I = 75.001* (which is given in Chen and Hwang (1992). The second solution is *R = 127.822 m* and *I = 129.971* (which is given in Easa (1994). The third solution is *R = 60.659 m* and *I = 155.028.* These three solutions are shown graphically in figure (2).

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Table (3) The results of a numerical example

Case number (1)	Given elements (2)	R (m) (3)	I (degrees) (4)
1	*T, E*	356.991	75.001
6	*T, M*	356.995 127.822 60.659	75.001 129.971 155.028
7	*E, Lc*	356.996	75.001
8	*T, Lc*	357.000	75.000
9	*E, M*	356.983	75.002
10	*M, Lc*	356.998	75.000

CONCLUSIONS

This paper presented a direct method for computing the radius of the horizontal circular curve R and the deflection angle between the tangents I using two of the other principal elements of the circular curve.

Three new direct cases (8), (9) and (10) and new direct formulas for cases (1), (6) and (7) are generated and applied to a numerical example of Chen and Hwang (1992). The proposed formulas gave the same results as Chen and Hwang (1992) and Easa (1994) for all cases except for case (6), the proposed formula gave three solutions.

Another research is needed to solve the rest cases in direct formulas.

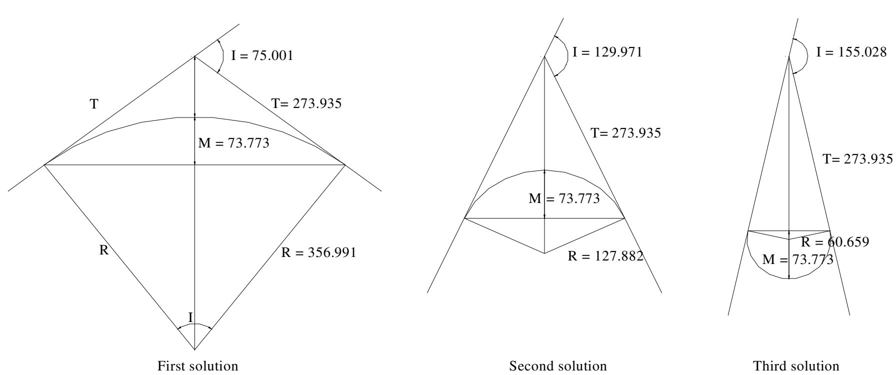

Fig. (2) Solutions for case (6)

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