

## A NEW APPROACH FOR 3D MEASUREING USING SINGLE IMAGE WITH A MIRROR IN CLOSE RANGE PHOTOGRAMMETRY

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### ملخص:

إن للمساحة التصويرية في المجال المحدود استخدامها العريضة في قياس الأبعاد الثلاثة للأهداف. و هذا يتطلب اخذ عدة صور للهدف الواحد من أماكن تصوير مختلفة وبجد ادن صورتين حتى يمكن عمل القياسات اللازمة لحساب الأبعاد الثلاثة له. و لقد تم استخدام الصورة المفردة لأخذ بعض القياسات للأهداف التي تظهر بها في البعدين و ذلك بعمل تقويم للصورة عن طريق استخدام عدد من نقاط التحكم الأرضية. أما في حالة أخذ القياسات في الثلاثة أبعاد من صورة مفردة ، فإنه يتوجب توفر شرط آخر لتحديد مواقع النقط في الفراغ. و هذا البحث يعرض طريقة مبتكرة لقياس الأبعاد الثلاثة للأهداف باستخدام صورة مفردة وذلك بالاستعانة بمرآة توضع في وضع يسمح بظهور الهدف وصورته المنعكسة من المرآة في صورة واحدة. تم وضع الأساس الرياضي لهذه الطريقة و تم اختبارها نظرياً و عملياً و أعطت نتائج واعده. هذا البحث تم اجراءه على الحالة الخاصة التي يكون فيها محور الكاميرا في الوضع الأفقي و مستوى المرآة في الوضع الرأسى.

### ABSTRACT

*The close-range photogrammetry has a wide use in measuring the three dimensions of the objects. This needs to have at least two photos for the object and for certain cases multi-images of the object may be needed. Single image has been used for measuring some dimensions of the objects, but the measurements must be in a plan (two dimensions) by making a rectification for the single image by using some of the ground control points. To make some three dimensions measurements from a single image, an additional condition must be found.*

*This paper presents a new approach for three dimensions measuring system by using a mirror that lies in a position to reflect the object and appear with it in the single image. The mathematical model of this approach has been developed and tested theoretically and practically and it gave promising results. This paper will concentrate on the special case that the camera axis is horizontal and the mirror plan is vertical.*

**KEYWORDS:** Close-range; Photogrammetry; Single-image; Mirror; 3D Measuring

### NOTATIONS

$X_i, Y_i$ and $Z_i$ :	Ground coordinates of the point,	$P_i$ :	The point in the real object,
$x'_i$ and $y'_i$ :	Photo coordinates of point in the real object,	$P'_i$ :	The image of point (P) on the mirror,
$x''_i$ and $y''_i$ :	Photo coordinates of point reflected through the mirror,	$\omega$ :	The angle between the mirror and film plane,
		$S$ :	The distance between the camera axis and the mirror in the film plane,
		$f$ :	The focal length of the camera.

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## INTRODUCTION

As we know that the close range photogrammetry has a wide use in several applications. Such applications are different in importance, some need high measurements accuracy and other do not, but most of them need three dimensions measurements. The three dimensions measurements in close range photogrammetry need at least two photos (Stereopairs) or in some cases multi-images of the object. The number of the images depend on the required accuracy and the object shape (Ebrahim, 1998). The stereopair is preferred even in photo interpretation because the photograph interpretation is more difficult with a single image compared to a stereo model as stated by (Allan L. and Holland D. 2000).

The use of the single image in close range photogrammetry has also its wide usage, but most of it in two dimensions. To measure from a single image in two dimensions, a rectification for the image must be done. Atkinson, 1996 has mentioned some of the used software in rectification such as Fotomass and Galilio Siscam. To have three dimensions measurements from a single image, an

additional condition must be found to determine the third dimension of the object. Matthias H, 1997 gave an overview over the different photogrammetric single image techniques, like digital rectification, unwrapping of parametric surfaces and differential rectification methods. Kraus K., 1993 mentioned also different methods of the digital rectification and differential rectification.

## THE APPROACH

### *Basic idea*

A new approach for three dimensions measurements using single image has been done and examined theoretically and practically by the means of the reflected object on a flat mirror. The idea of this approach is the line passing through the camera lens and the image of a point will be the first direction to the space point. The reflected line from the mirror for the line passing through the camera lens and the image of the point from the mirror will be the second direction of the space point. The intersection of these two lines will determine the point position in the space.

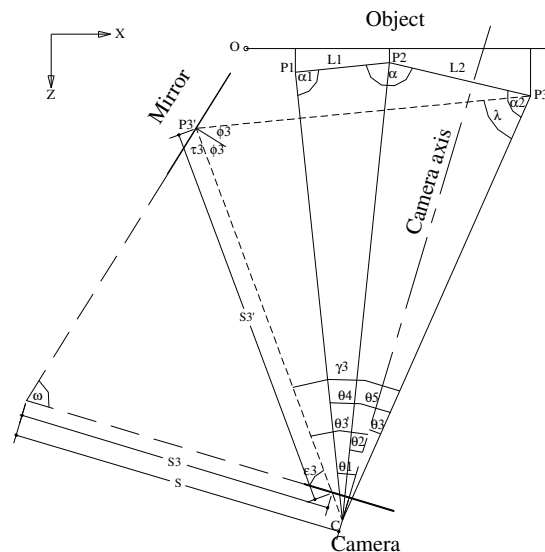


Figure (1): The geometry of the object, mirror and camera.

### Details of computations

#### Step 1: Computing the coordinates of the camera

The resection method which is a way of determining the position of an unknown point by occupying the point and measuring the horizontal angles between at least three control points (Moffitt F. and Boucherd H. 1987) was used to determine the coordinates of the camera. The horizontal angles at the camera station between the control points are calculated -using the photo measurements- instead of measuring in the conventional method. Referring to figure (1) the following equations can be driven:

$$\theta_1 = \tan^{-1} \frac{x'_1}{f} \quad (1)$$

$$\theta_2 = \tan^{-1} \frac{x'_2}{f} \quad (2)$$

$$\theta_3 = \tan^{-1} \frac{x'_3}{f} \quad (3)$$

$$\theta'_3 = \tan^{-1} \frac{x''_3}{f} \quad (4)$$

$$\theta_4 = \theta_1 - \theta_2 \quad (5)$$

$$\theta_5 = \theta_2 - \theta_3 \quad (6)$$

$$\theta_6 = \theta_1 - \theta_3 \quad (7)$$

$$L_1 = \sqrt{(X_2 - X_1)^2 + (Z_2 - Z_1)^2} \quad (8)$$

$$L_2 = \sqrt{(X_3 - X_2)^2 + (Z_3 - Z_2)^2} \quad (9)$$

$$A_{L1} = \tan^{-1} \frac{Z_1 - Z_2}{X_1 - X_2} \quad (10)$$

$$A_{L2} = \tan^{-1} \frac{Z_3 - Z_2}{Z_3 - Z_2} \quad (11)$$

$$\alpha = A_{L1} - A_{L2} \quad (12)$$

$$\eta = \alpha_1 + \alpha_2 = 360 - (\alpha + \theta_4 + \theta_5) \quad (13)$$

$$\cot \alpha_1 = \cot \eta + \frac{L_1 \cdot \sin \theta_5}{L_2 \cdot \sin \theta_4 \cdot \sin \eta} \quad (14)$$

Restitution in equation (13) by the value of  $\alpha_1$ ,  $\alpha_2$  can be computed.

Using one of the control points (e.g.  $P_1$ ) the following relationships can be driven:

$$\alpha_3 = 180 - (\alpha_1 + \theta_4) \quad (15)$$

$$CP_1 = \frac{L_1 \cdot \sin \alpha_3}{\sin \theta_4} \quad (16)$$

$$A_{PC} = A_{L1} \pm 180 + \alpha_1 \quad (17)$$

$$X_C = X_1 + CP_1 \cdot \cos A_{PC} \quad (18)$$

$$Z_C = Z_1 + CP_1 \cdot \sin A_{PC} \quad (19)$$

$$Y_C = Y_1 - \frac{y'_1 \cdot CP_1}{\sqrt{f^2 + x_1'^2}} \quad (20)$$

$$A_{CC} = A_{PC} \pm 180 + \theta_1 \quad (21)$$

(8) Step 2: Computing the distance  $S$  between the camera axis and the mirror in the film plane.

The distance ( $S$ ) is the key factor for determining the distance between the camera and any object point which used to determine the point coordinates. To

determine the distance (S) –in the studied case- the angle ( $\omega$ ) between the mirror and film planes must be known.

Using one of the control points (e.g.  $P_3$ ) the following relationships can be driven:

$$\varepsilon_3 = 90 - \theta'_3 \quad (22)$$

$$\tau_3 = 180 - (\omega + \varepsilon_3) \quad (23)$$

$$\phi_3 = 90 - \tau_3 \quad (24)$$

$$\gamma_3 = \theta_3 - \theta'_3 \quad (25)$$

$$\lambda_3 = 180 - (2\phi_3 + \gamma_3) \quad (26)$$

$$CP'_3 = \frac{CP_3 \cdot \sin \lambda_3}{\sin(2\phi_3)} \quad (27)$$

$$S'_3 = CP'_3 - \frac{f}{\cos \theta'_3} \quad (28)$$

$$S_3 = \frac{S'_3 \cdot \sin \tau_3}{\sin \omega} \quad (29)$$

$$S = S_3 - x''_3 \quad (30)$$

*Step 3: Computing the points coordinates*

As a reverse manner of computing the distance (S), the distance from the camera to any point can be computed then the points' coordinates are computed using the following equations.

$$\phi_i = \omega - \theta'_i \quad (31)$$

$$\tau_i = 90 - \phi_i \quad (32)$$

$$S_i = S + x''_i \quad (33)$$

$$S'_i = \frac{S_i \cdot \sin \omega}{\sin \tau_i} \quad (34)$$

$$CP'_i = S'_i + \frac{f}{\cos \theta'_i} \quad (35)$$

$$\lambda_i = 180 - (2\phi_i + \gamma_i) \quad (36)$$

$$CP_i = \frac{CP'_i \cdot \sin(2\phi_i)}{\sin \lambda_i} \quad (37)$$

$$A_{CP_i} = A_{CC} - \theta_i \quad (38)$$

$$X_i = X_C + CP_i \cdot \cos A_{CP_i} \quad (39)$$

$$Z_i = Z_C + CP_i \cdot \sin A_{CP_i} \quad (40)$$

$$Y_i = Y_C + \frac{y'_i \cdot CP_i}{\sqrt{f^2 + x_i'^2}} \quad (41)$$

## EXPERIMENTAL WORK

The approach has been tested theoretically by using a solved example to verify the mathematical model. To verify the accuracy of the theoretical approach, a practical test has been done. A horizontal single image was taken for a control field with its reflected image on a vertical mirror to compare the ground coordinates of the points computed using the new approach to the real ground coordinates of the control field.

Also five photos for the control field were taken from different camera positions to obtain the accuracy achieved from such system by using multi-images technique. The control field is a part of a big movable control field which designed specially for photogrammetric purposes (Ebrahim, 1992). The used part contains 25 control points arranged in columns and rows on its body (100 x 200) cm. The control points are 20 cm apart and their height range from 0 to 20 cm. The used mirror was flat one of (90 x 60) cm. The position of the mirror was chosen so that the whole control field appears in the mirror as shown in figure (2).

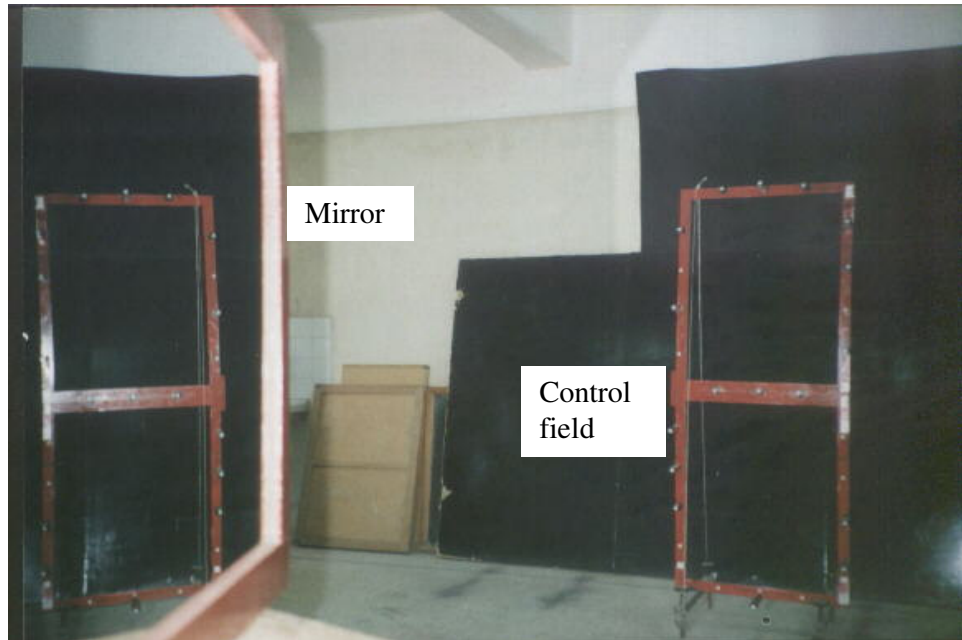


Figure (2) The control field and the mirror positions.

A Total Station TOPCON 712 has been used to determine the real ground coordinates of the points on the control field and the ground coordinates of the top ends of the mirror by using the base line method. The base line length was 5 m and it was 5.5 m away from the control field. The horizontal directions to each control point were measured from both ends of the base line while the vertical angle to each point was measured from one end only. A non-metric camera (YASHICA zoomtec 90 super) has been used for photographing in this research. The focal length of the camera was fixed to 38 mm. Roll film (FUJI 100 ASA) with 24 by 36 image format has been used. The photos have been converted to the digital form using a flatbed scanner (MUSTEK scanExpress 600 SEP). The position of the camera related to the mirror is shown in figure (3).

## THE RESULTS AND ANALYSIS

### *The Accuracy*

The accuracy is an important factor in the world of measurements. So, it has been

defined in several textbooks and articles. As example, Edward M and Gordon Gracie, 1981 defined the accuracy as the degree of conformity or closeness of a measurement to the true value. They state that accuracy includes not only the effects of random errors but also any bias due to uncorrected systematic errors. If there is no bias, the standard deviation can be used as a measure of accuracy.

Buckner R.B., 1983 defined accuracy as the degree of conformity with a standard ("the truth"). He mentioned that the accuracy relates to the quality of a result, and is distinguished from precision, which relates to the quality of the operation by which the result is obtained.

Kenneth E. and Donald J., 1995 defined the accuracy as the degree to which information on a map or in digital database matches true or accepted values. Accuracy is an issue pertaining to the quality of data and number of errors contained in a data set or map.

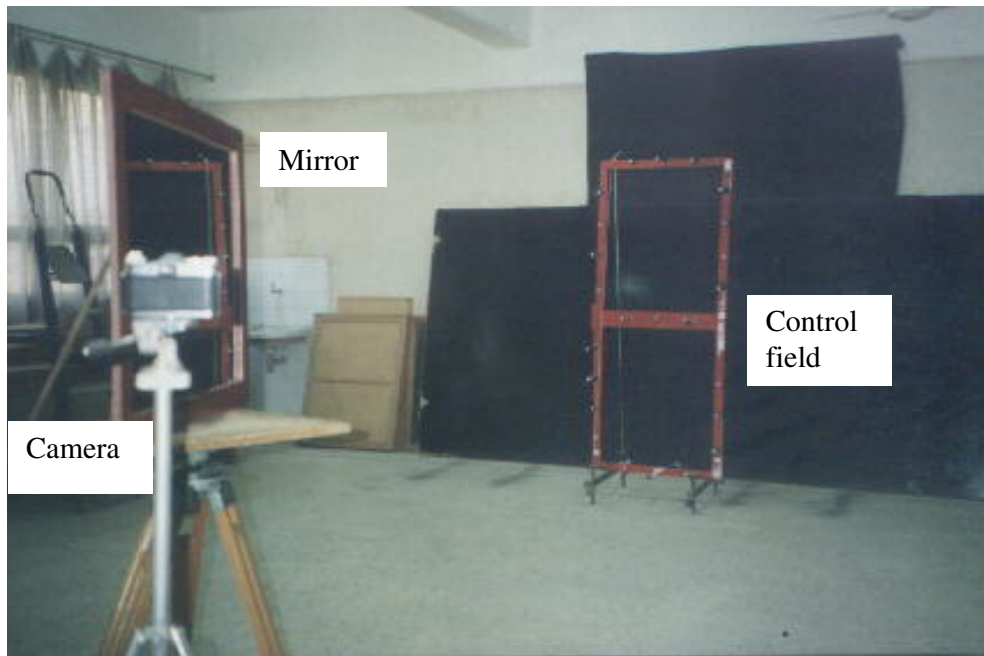


Figure (3) The position of the camera related to the control field and the mirror.

In theory of errors of measurements the term accuracy is traditionally used for a concept that measures the closeness of derived estimated or predicated data to reality. (Ebrahim, 1992)

The accuracy can be evaluated by using one of the following two methods (Hottier, 1976):

- 1- Check measurements
- 2- The accuracy predictor

In the first method, the photogrammetric results are compared with the results obtained from a more accurate measuring procedure. The accuracy predictor is a theoretical way to evaluate a system using its main parameters.

Fraser, 1990 agree with Hottier, 1976 for the first method and mentioned that “If check points are available in the object space, the root-mean square (RMS) errors of photogrammetrically determined target

point co-ordinates can be used as an accuracy measure”.

Photogrammetrists very often estimate the accuracy of a method by controlled experiments, where the photogrammetrically determined co-ordinates are compared with so called given co-ordinates that have an accuracy that considerably higher than that of the method to be checked (Ebrahim, 1992).

### ***The results***

The photo coordinates  $x'_i$ ,  $y'_i$  and  $x''_i$  of the points are measured. Three points are used as control points to calculate the coordinates of the camera and the direction of its axis and 19 points are used as checkpoints. The angle ( $\omega$ ) is computed from the ground measurements of the mirror ends and the computed camera axis direction. The ground coordinates of the checkpoints are calculated using the photo measurements and the coordinates of the

camera with the help of a computer program developed for this research. The check measurement method was used to test the accuracy of the new approach. The

differences in the ground coordinates between the real one and that computed using the new approach are shown in table(1).

Table (1): The errors in points' coordinates

Point	$\Delta X$ (cm)	$\Delta Y$ (cm)	$\Delta Z$ (cm)
1	-1.387	-0.366	-4.173
2	0.460	0.132	-2.888
3	-0.795	0.081	-1.165
4	-0.439	0.174	0.620
6	-0.823	0.291	-3.935
9	-1.251	0.791	1.733
10	-0.800	0.561	2.177
11	-0.561	-0.264	-1.822
12	-0.322	-0.527	-2.906
13	-0.196	-0.724	-0.984
14	-0.048	-0.469	-3.980
15	-0.312	-0.450	-4.460
16	-0.360	-0.777	-1.677
17	-0.296	2.211	4.364
18	-1.042	2.124	4.200
19	-1.950	-0.129	-5.870
20	-0.006	0.030	-0.030
21	-0.233	-0.128	-1.369
22	0.186	0.098	1.372

The standard deviations in X, Y and Z directions are:

$$\sigma X = 0.584 \text{ cm}$$

$$\sigma Y = 0.823 \text{ cm}$$

$$\sigma Z = 2.926 \text{ cm}$$

### ***The Analysis***

These values of the standard deviation are acceptable specially for the architectural application of the close range photogrammetry. The big value of ( $\sigma Z$ ) may be due to the position of the mirror which affect the intersection angle ( $\lambda$ ). When this angle is small (the mirror is close to the camera), the error in (Z) direction will be of great value because the direct line from the camera to the point and

the reflected one will not intersect in an exact point. The results are coincides with the known phenomena in photogrammetric works that the lowest accuracy comes always in the camera axis direction (Z). The coating layer of the mirror was in the backside and this allows the rays to be refracted through the thickness of the mirror and may cause an error in (Z) direction too.

The accuracy obtained from the multi-images technique was as follows:

$$\sigma X = 0.457 \text{ cm}$$

$$\sigma Y = 0.773 \text{ cm}$$

$$\sigma Z = 0.547 \text{ cm}$$

From these results, it is clear that the accuracy obtained from the new approach is almost the same like the multi image

technique unless in Z direction which may be because of the previous mention reasons.

## CONCLUSION

A new three dimensions measuring by using single image approach has been developed. The approach has been tested and proved practically. The obtained results of this research are promising. It can be concluded that it is possible to get the three dimensions of the objects using a single image with the aid of a mirror reflect the object in the same single image. The accuracy of the obtained results is acceptable for the architecture application and can be improved by using a high quality mirror in a position near to the object to achieve big intersection angles at the object. This will improve the accuracy of the measurements in Z direction.

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